Part A (Multiple choice) Each problem worth 9 points

1. Find the general solution of the equation

$$y^{(4)} + 2y'' + y = 0.$$

a. $c_1 \cos x + c_2 \sin x$

b. $c_1 e^{x} + c_2 e^{-x} + c_3 \cos x + c_4 \sin x$

c. $c_1 + c_2 x + c_3 \cos x + c_4 \sin x$

d. $c_1 \cos x + c_2 \sin x + c_3 x \cos x + c_4 x \sin x$

e. $c_1 e^{x} + c_2 x e^{x} + c_3 e^{-x} + c_4 x e^{-x}$

2. Which of the following is a solution of

$$y^{(4)} - 8y' = 0 ?$$

a. $y = 2t - 1$

b. $y = e^{-2t} \cos 2t$

c. $y = e^{t} \cos \sqrt{3}\, t$

d. $y = e^{-t} \sin \sqrt{3}\, t$

e. $y = \sin \sqrt{3}\, t$

3. The characteristic equation for the differential equation

$$y^{(4)} + y''' - 2y'' = x + e^x$$

is $r^2 (r - 1) (r + 2) = 0$. Find the form of a particular solution.

a. $Ax + Bx^2 + Ce^x$

b. $Ax^3 + Cxe^x$

c. $Ax^3 + Bx^2 + Cxe^x$

d. $Ax + Bx^2 + Cxe^x$

e. $Ax^3 + Bx^2 + Ce^x$

4. The Laplace transform of $f(t) = 1 + u_2(t) - 3\,\delta(t - 2)$ is

a. $1 + e^{-2s}/s - 3\,e^{-2s}$

b. $1/s + e^{-2s}/s - 3e^{-2s}$

c. $1/s - 2\,e^{-2s}$

d. $1/s + e^{-2s} - 3$

e. $1 + e^{-2s} - 3e^{2s}$

5. The inverse Laplace transform of $F(s) = \dfrac{e^{-s}}{s^2 + 2s + 2}$ is

a. $e^{-t} \sin(t + 1)$

b. $e^{t} \sin(t - 1)$

c. $u_1(t)\ e^{t-1} \sin(t - 1)$

d. $u_1(t)\ e^{-(t-1)} \sin(t - 1)$

e. $u_1(t)\ e^{t+1} \sin(t + 1)$

6. Using the convolution theorem, find the inverse Laplace transform of

$$F(s) = \frac{1}{s^4 (s^2 + 1)} \quad .$$

a. $\frac{1}{6}\ t^3 \sin t$

b. $\frac{1}{6}\ t^3 + \sin t$

c. $\int_o^t (t + \tau)^3 \sin \tau \, d\tau$

d. $\frac{1}{6} \int_o^t (\square - \tau)^3 \sin \tau \, d\tau$

e. $\frac{1}{6} \int_o^t (t + \tau)^3 \sin \tau \, d\tau$

7. Evaluate $$\int_0^\infty \delta\left(t - \frac{\pi}{4}\right) \frac{\cos^2 t}{1 + \sin^2 t}\ t\ dt.$$

a. $\frac{\pi}{12}$

b. $\frac{\pi}{6}$

c. $\frac{\pi}{4}$

d. $\frac{\pi}{3}$

e. $\frac{\pi}{2}$

8. Solve the initial value problem $y'' + 4y = \delta(t - 1),\ y(0) = 0,\ y'(0) = 0.$

a. $\frac{1}{2}\ u_1(t)\ \sin 2(t - 1)$

b. $\frac{1}{2}\ u_1(t)\ \sin 2t$

c. $u_1(t)\ \sin 2t$

d. $u_1(t - 1)\ \sin 2(t - 1)$

e. $\frac{1}{2}\ e^{-t}\ \sin 2(t - 1)$

PART B (partial credit)

9. Consider the equation $y''' - y'' = e^x/x$.

a. In the method of variation of parameters, one considers a particular solution of the form $u_1(x) + u_2(x)\,x + u_3(x)\,e^x$.

Write down the equations satisfied by u_1', u_2' and u_3'.

b. Find u_3 .

u_3 = ______________________

10. Solve the initial value problem $y'' + 4y = u_{\pi}(t)$,

$y(0) = 0$, $y'(0) = 1$.

a. Find the Laplace transform of y.

Ò {y} = ______________________________

b. Find y.

y = ______________________________