1. Set $x_1 = y$ and $x_2 = y'$ in the equation $y'' - 4y' + 2ty = e^t$ to obtain a system of two first order equations. Which of the following equations is satisfied by the vector

$$x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \quad ?$$

a. $x' = \begin{pmatrix} 0 & -1 \\ 2t & 4 \end{pmatrix} x + \begin{pmatrix} 0 \\ e^t \end{pmatrix}$

b. $x' = \begin{pmatrix} 0 & 1 \\ -2t & 4 \end{pmatrix} x + \begin{pmatrix} 0 \\ e^t \end{pmatrix}$

c. $x' = \begin{pmatrix} 0 & 1 \\ -2t & 4 \end{pmatrix} x + \begin{pmatrix} e^t \\ 0 \end{pmatrix}$

d. $x' = \begin{pmatrix} 1 & 0 \\ -2t & 4 \end{pmatrix} x + \begin{pmatrix} 0 \\ e^t \end{pmatrix}$

e. $x' = \begin{pmatrix} 0 & -1 \\ 2t & -4 \end{pmatrix} x + \begin{pmatrix} 0 \\ e^t \end{pmatrix}$

2. The inverse of the matrix $\begin{pmatrix} -1 & i \\ i & 2 \end{pmatrix}$ is

a. $\begin{pmatrix} 1 & -i \\ -i & 2 \end{pmatrix}$

b. $\begin{pmatrix} -2 & i \\ i & 1 \end{pmatrix}$

c. $\begin{pmatrix} 2 & -i \\ -i & 1 \end{pmatrix}$

d. $\begin{pmatrix} 1 & i \\ 2i & 2 \end{pmatrix}$

e. inverse does not exist

3. The Wronskian determinant $W(X,Y,Z)$, where

$$X = \begin{pmatrix} t \\ t \\ t^2 \end{pmatrix} \quad , \quad Y = \begin{pmatrix} 0 \\ t \\ 0 \end{pmatrix} \quad , \quad Z = \begin{pmatrix} t \\ 0 \\ t^2 \end{pmatrix} \quad , \quad \text{is}$$

a. 0 b. $t^4 - 1$ c. $t - t^4$ d. $2t^4$ e. $1 - t$

4. The general solution of the system $x' = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} x$

is $x(t) = c_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} e^{-t} + c_3 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{-t}$.

There is a unique solution $x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$ such that $x(0) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$. This solution has component $x_3(t) =$

a. $x_3 = \frac{1}{3} e^{2t} + \frac{2}{3} e^{-t}$ b. $x_3 = \frac{2}{3} e^{2t} + \frac{1}{3} e^{-t}$ c. $x_3 = \frac{4}{3} e^{2t} - \frac{1}{3} e^{-t}$

d. $x_3 = -\frac{1}{3} e^{2t} + \frac{4}{3} e^{-t}$ e. $x_3 = e^{-t}$

5. The general solution of the system $x' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} x$ is

a. $c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t}$ b. $c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t}$

c. $c_1 \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} + c_2 \begin{pmatrix} \sin t \\ \cos t \end{pmatrix}$

d. $c_1 \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} + c_2 \begin{pmatrix} \sin t \\ -\cos t \end{pmatrix}$

e. $c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t}$

6. Find the fundamental matrix $\Phi(t)$ of the system $x' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} x$ such that $\Phi(0) = I$.

a. $\begin{pmatrix} \cos t & -\sin t \\ -\sin t & \cos t \end{pmatrix}$

b. $\begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix}$

c. $\begin{pmatrix} \cos t & \sin t \\ -\sin t & -\cos t \end{pmatrix}$

d. $\begin{pmatrix} -\cos t & -\sin t \\ \sin t & \cos t \end{pmatrix}$

e. $\begin{pmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{pmatrix}$

7. The eigenvalues of the matrix $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$ are

a. 2, 2, 3 b. 1, − 2, 3 c. 2, 2, 2 d. − 1, 2, 3 e. 1, 2, 3

8. Find the solution of the intial value problem

$$x' = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} x \quad ; \quad x(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

a. $\begin{pmatrix} 2 \\ 2 \end{pmatrix} te^{2t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{2t}$

b. $\begin{pmatrix} 1 \\ -1 \end{pmatrix} te^{2t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{2t}$

c. $-\begin{pmatrix} 1 \\ 1 \end{pmatrix} t\,e^{2t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{2t}$

d. $\begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{2t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{2t}$

e. $\begin{pmatrix} 1 \\ 1 \end{pmatrix} t\,e^{2t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

9. PARTIAL CREDIT

a. Find the general solution of the homogeneous system $x' = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix} x$.

b. Find a particular solution of the non-homogeneous system

$$x' = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad .$$

10. (a) Solve the initial value problem $x' = \begin{pmatrix} -1 & 2 \\ -2 & -1 \end{pmatrix} x \; ; \; x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

answer: $x =$ ____

(b) Find $\lim_{t \varnothing \infty} x(t)$

answer: $\lim_{t \varnothing \infty} x(t) =$ ____