1. Which of the sketches below best describes the trajectories of the system $x' = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} x$?

a. I b. II c. III d. IV e. V

2. A certain system $x' = Ax$ with A a 2x2 constant matrix has r=2 as an eigenvalue of multiplicity two, only one eigenvector $\xi = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and a generalized eigenvector $\eta = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$. Which of the sketches below best describes the trajectories of the system?

a. IV b. V c. VI d. VII e. VIII

3. Consider the system $x' = \begin{bmatrix} -3 & 2 \\ 0 & -1 \end{bmatrix} x$.

The origin (0,0) is a critical point of the following type:

a. Proper node, asymptotically stable

b. Center, stable

c. Proper node, unstable

d. Saddle point, unstable

e. Improper node, asymptotically stable

4. The system $x' = (x - 1)\, y\, (y + 1)$
$y' = x\, (y + 2)$

has the following critical points:

a. (0, 0), (1, 0) and (1, 2)

b. (0, 1) and (0, –1) only

c. (0, 0), (0, –1) and (1, –2)

d. (0, 0) and (0, –1) only

e. (0, 0), (0, –2) and (1, 0)

5. The critical point $(\pi, \frac{\pi}{2})$ of the system

$$x' = y \sin x$$
$$y' = y - x^2 \cos y - \frac{\pi}{2}$$

is:

a. Center, stable

b. Saddle point, unstable

c. Proper node, asymptotically stable

d. Improper node, asymptotically stable

e. Spiral point, unstable

6. The critical point (0,0) of the system

$$x' = x + 3y + x^2$$
$$y' = -3x + y + y^2$$

is:

a. Spiral point, unstable

b. Center or Spiral point, stability indeterminate

c. Proper node, unstable

d. Spiral point, asymptotically stable

e. Saddle point, unstable

7. Consider the initial value problem

$$\frac{dy}{dt} = 1 - t + y$$

$$y(0) = 1 \ .$$

The Euler method with step size $h = 0.1$ gives the approximate value of the solution at $t = 0.2$ as

a. 1.40 b. 1.41 c. 1.42 d. 1.43 e. 1.44

8. Consider the initial value problem $\frac{dy}{dt} = t + 2y$, $y(0) = 1$

The *improved* Euler method with step size $h = \frac{1}{2}$ gives the approximate value of the solution at $t = \frac{1}{2}$ as

a. $\frac{13}{8}$ b. $\frac{15}{8}$ c. $\frac{17}{8}$ d. $\frac{19}{8}$ e. $\frac{21}{8}$

9. Consider the system $x' = y - x e^y$, $y' = -2x - y e^y$

which has a critical point at (0,0).

Use Liapunov's method to prove that the critical point, (0, 0), of the above system is asymptotically stable. Give the Liapunov function which you use in the place indicated below. In addition indicate clearly how Liapunov's theorem is used in conjunction with your chosen Liapunov function.

Answer V(x,y) =_____

10. Find the solution of the heat conduction problem

$2\, u_{xx} = u_t \quad , \quad 0 < x < 3 \, , \quad t > 0 \, ;$

$u\,(0,t) = 0 \quad , \; u\,(3,t) = 0 \quad , \; t > 0 \, :$

$u\,(x, 0) = 2 \sin \frac{\pi x}{3} \; - \; \sin \pi x \; + \; 3 \sin 2\pi x.$