1. Find the inverse Laplace transform of $\frac{s - 2}{s^2 - 4s + 13}$ .

a. $e^{2t} \sin 3t$

b. $e^{2t} \cos 3t$

c. $e^{-2t} \sin 3t$

d. $e^{-2t} \cos 3t$

e. $e^{3t} \cos 2t$

2. Find the Laplace transform of $f(t)\,\delta(t - 1)$.

a. 1

b. $e^{-s} f(s)$

c. $e^{-s} f(1)$

d. $e^{s} f(1)$

e. $f(1)$

3. Let $f(t) = \begin{cases} 0 & 0 \le t < 2 \\ t-2 & 2 \le t < 4 \\ 0 & 4 \le t < \bullet \end{cases}$ then, using the unit step functions, f (t) can be rewritten as

a. $(t - 2)\, u_2(t)$ b. $u_2(t)\, t$ c. $(t - 2)\, (u_2(t) + u_4(t))$

d. $(u_4(t) - u_2(t))\,(t-2)$ (e) $(u_2(t) - u_4(t))\,(t-2)$

4. The solution of the initial value problem $y'' + 2y' + y = \delta(t)$, $y = 0$, $y'(0) = 2$ is

a. $-3\, te^{-t}$ b. $3te^{-2t}$ c. $3te^{t}$

d. $3te^{-t}$ e. $3t^2\, e^{-t}$

5. The general solution of the equation $y^{iv} - y = 0$ is

a. $c_1 e^t + e_2\, te^t + c_3 \sin t + c_4 \cos t$

b. $c_1 t + c_2\, e^{-t} + c_3 \sin t + c_4 \cos t$

c. $c_1\, e^{-t} + c_2\, e^t + c_3 e^{2t} + c_4\, e^{-2t}$

d. $c_1\, e^{-t} + c_2 e^{2t} + c_3 \cos 2t + c_4 \sin 2t$

e. $c_1 e^t + c_2 e^{-t} + c_3 \cos t + c_4 \sin t$

6. Find a particular solution of the non-homogeneous equation

$$y^{iv} + y'' = x^2$$

a. $\frac{1}{6}\, x^3 + x^2 - x$

b. $\frac{1}{12}\, x^4 + 2\, x^3 - x^2$

c. $\frac{1}{15}\, x^5 + \frac{1}{12}\, x^4 - \frac{1}{6}\, x^3$

d. $\frac{1}{12}\, x^4 - x^2$

e. $2x^3 - \frac{1}{4}\, x^2 + x - 2$

7. The matrix $A = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ has $r = 2$ as a repeated eigenvalue of multiplicity 2.

Find the eigenvectors of A corresponding to this eigenvalue.

a. $c_1 \begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix}$

b. $c_1 \begin{pmatrix} -4 \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

c. $c_1 \begin{pmatrix} -1 \\ 4 \\ 0 \end{pmatrix}$

d. $c_1 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

e. $c_1 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$

8. Find the general solution of $x' = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix} x$.

a. $c_1 \begin{pmatrix} \cos t + \sin t \\ \cos t \end{pmatrix} + c_2 \begin{pmatrix} \cos t + \sin t \\ \sin t \end{pmatrix}$

b. $c_1 \begin{pmatrix} \cos t \\ \cos t + \sin t \end{pmatrix} + c_2 \begin{pmatrix} -\sin t \\ \cos t - \sin t \end{pmatrix}$

c. $c_1 \begin{pmatrix} \cos t \\ \cos t - \sin t \end{pmatrix} + c_2 \begin{pmatrix} \sin t \\ \cos t + \sin t \end{pmatrix}$

d. $c_1 \begin{pmatrix} \cos t - \sin t \\ \cos t \end{pmatrix} + c_2 \begin{pmatrix} \cos t + \sin t \\ \sin t \end{pmatrix}$

e. $c_1 \begin{pmatrix} \cos t + \sin t \\ \cos t \end{pmatrix} + c_2 \begin{pmatrix} \cos t - \sin t \\ -\sin t \end{pmatrix}$

9. The eigenvalues of the matrix $A = \begin{pmatrix} 2 & -1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ are

a. $2,\ 2 + i,\ 2 - i$

b. $2, 2, 2$

c. $2, 2, -1$

d. 2, 1, – 1 e. 2, 1 + 2i, 1 – 2i

10. The solution of the initial value problem

$$x' = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{is}$$

a. $\begin{bmatrix} -1 \\ 1 \end{bmatrix} t e^{2t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-2t}$

b. $\begin{bmatrix} -1 \\ 1 \end{bmatrix} te^{2t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{2t}$

c. $\begin{bmatrix} 1 \\ -1 \end{bmatrix} te^{2t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{2t}$

d. $\begin{bmatrix} 1 \\ -1 \end{bmatrix} te^{-2t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{2t}$

e. $\begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{2t}$

11. The trajectory corresponding to the solution

of
$$\begin{cases} \frac{dx}{dt} = 4y, & x(0) = 2 \\ \frac{dy}{dt} = x, & y(0) = 0 \end{cases}$$
is

a. $x^2 + y^2 = 4$ b. $\frac{x^2}{4} + \frac{y^2}{16} = 1$ c. $\frac{x^2}{4} + y^2 = 1$

d. $x^2 + 2y^2 = 4$ e. $\frac{x^2}{2} + y^2 = 2$

12. Consider the system

$$\frac{dx}{dt} = x + y^2$$
$$\frac{dy}{dt} = x + y\ .$$

a. The critical point (0, 0) is a center

b. The critical point (0, 0) is asymptotically stable

c. The critical point (0, 0) is an unstable spiral point

d. The critical point (–1, 1) is an unstable improper node

e. The critical point (–1, 1) is a saddle point

13. The system $\frac{dx}{dt} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} x$

has a critical point at the origin. Which of the pictures below best describes the trajectories of this system near the origin?

a. II b. IV c. VI d. VIII e. I

14. The system $\frac{dx}{dt} = \begin{bmatrix} 1 & 2 \\ -5 & -1 \end{bmatrix} x$

has a critical point at the origin. Which of the pictures below best describes the trajectories of this system near the origin?

a. I b. III c. V d. VII e. II

15. Consider the initial value problem $\begin{cases} y' = t + 2y \\ y(0) = 1 \end{cases}$

Use Euler's method with step size h = 0.1 to find the approximate value of the solution at t = 0.2.

a. 1.41 b. 1.42 c. 1.43 d. 1.44

e. 1.45

16. Consider the functions

$$f(x) = \sin x \cos x,\ \ g(x) = x^3 + \cos 3x, \qquad h(x) = 1 + x^2 + \cos 2x$$
$$k(x) = x + \tan 4x,\ \ \square(x) = (x + 1)^3 \sin 5x.$$

a. Only 2 of these functions are even

b. Only 1 of these functions is odd

c. Only 2 of these functions are neither even nor odd

d. Only 3 of these functions are odd

e. Only 3 of these functions are even

17. If the function $f(x) = x,\ 0 \le x \le \square$ is expanded in a Fourier Sine Series

$$\sum_{n=1}^{\bullet} b_n \sin \frac{n\pi x}{\square}\ ,$$ find the value of $b_n$.

a. $\frac{\square}{n\pi}(-1)^{n}$

b. $\frac{\square}{n\pi}(-1)^{n+1}$

c. $\frac{2\square}{n\pi}(-1)^{n}$

d. $\frac{2□}{n\pi}(-1)^{n+1}$ e. 0

18. Consider a uniform rod of length $\pi$ with an initial temperature given by sin x. Assume that both ends of the bar are insulated. What is the steady state temperature as $t \to \infty$ ?

a. $\frac{2}{\pi}$ b. $\frac{3}{\pi}$ c. 0 d. $\frac{-2}{\pi}$ e. $\frac{-3}{\pi}$

19. Let $f(x) = \begin{cases} 1, & 0 \le x < \frac{1}{2} \\ -1, & \frac{1}{2} \le x < 1 \end{cases}$

and extend f to be an odd periodic function with period 2. The Fourier series of f(x) converges at $x = 0$ to

a. 1 b. 0 c. $-1$ d. $\frac{1}{2}$ e. $-\frac{1}{2}$

20. A metal rod of unit length and heat constant $\alpha = 1$ is initially at temperature $100^{\circ}$ C throughout with its end points maintained at $0^{\circ}$ C for all time. Find the temperature u (1/2,t) of the middle of the rod at any time t.

a. $$\frac{400}{\pi} \sum_{k=1}^{\infty} \frac{1}{2k-1} e^{-(2k-1)^2\pi^2 t} (-1)^{k+1}$$

b. $$\frac{200}{\pi} \sum_{k=1}^{\infty} \frac{1}{2k-1} e^{-(2k-1)^2\pi^2 t} (-1)^{k+1}$$

c. $\frac{400}{\pi} \sum_{k=1}^{\bullet} \frac{1}{2k-1} e^{-(2k-1)^2\pi^2 t} (-1)^k$

d. $\frac{200}{\pi} \sum_{k=1}^{\bullet} \frac{1}{2k-1} e^{-(2k-1)^2\pi^2 t} (-1)^k$

e. 100 $^o$C for all time.