MATH 325 PRACTICE Test I

Part A (multiple choice) each problem worth 9 points

1. Find the general solution of the equation
$$y''' - 3y'' + 4y' - 2y = 0 .$$

a. $C_1 e^x + C_2 e^x \cos x + C_3 e^x \sin x$

b. $C_1 e^x + C_2 x e^x + C_3 x^2 e^x$

c. $C_1 + C_2 \cos x + C_3 \sin x$

d. $C_1 e^{-x} + C_2 e^{-x} \cos x + C_3 e^{-x} \sin x$

e. $C_1 x \cos x + C_2 x \sin x + C_3 e^{-x}$

2. Determine a suitable form for a particular solution of

$$y'' - y' = x e^x$$
using the method of undetermined coefficients.

a. $A + B e^x$

b. $A e^x + B x e^x$

c. $A x + B e^x$

d. $A x^2 e^x + B x e^x$

e. $A + B x e^x$

3. Let f(t) be the function defined by $\quad f(t) = \begin{cases} 1, & t < 1 \\ 2, & t \geq 1 \end{cases}$.

From the definition of the Laplace transform, or otherwise, find the Laplace transform F(s) of f(t).

a. $\dfrac{1 + e^s}{s}$

b. $\dfrac{e^s - 1}{s}$

c. $\dfrac{e^{-s} + 1}{s}$

d. $\dfrac{e^s - e^{-s}}{s}$

e. $\dfrac{e^s + e^{-s}}{s}$

4.   Find the inverse Laplace transform of the function

$$F(s) = \frac{1}{s^2 - 2s + 5} + 2\frac{e^{-(s+1)}}{s + 1}$$

a. $e^t \cosh t - u_1(t)(t-1)$

b. $e^t \cos 2t + e^{-t} u_1(t)$

c. $\frac{1}{2} e^{-t} \sin 2t + u_1(t)(t-1)$

d. $\frac{1}{2} e^t \sinh t + e^{-t} u_1(t)$

e. $\frac{1}{2} e^t \sin 2t + 2 e^{-t} u_1(t)$

5.   Using the definition, compute the Laplace transform of

$$f(t) = \delta(t-1) u_{\frac{1}{2}}(t) e^{t-1} \quad .$$

a. $u_{\frac{1}{2}}(s) + 1$

b. $e^{-(s+1)}$

c. e^{-s}

d. $\delta(s)$

e. 0

6.   The following form of the general solution of the non-homogeneous equation $y'' + y' - 6y = x$ is found using the method of undetermined coefficients. Which one is it?

a. $C_1 e^{-3x} + C_2 e^{2x} + x + \frac{1}{36}$

b. $C_1 e^{-3x} + C_2 e^{2x} - \frac{1}{6}\left(x + \frac{1}{6}\right)$

c. $C_1 (e^{-3x} + e^{2x}) - C_2 x + \frac{1}{6}$

d. $C_1 e^{2x} + C_2 x e^{2x} + e^{-3x}$

e. $C_1 e^{-3x} + C_2 x e^{-3x} + e^{2x}$

7.   Find the solution of the intial value problem

$$y'' + 3y = \cos t$$

$$y(0) = 0,\ y'(0) = 0 \ .$$

a. $\dfrac{1}{2}\cos t - \dfrac{1}{2}\cos\sqrt{3}\ t$

b. $1 - \cos 2t$

c. $\sin\sqrt{3}\ t + \cos t$

d. $\dfrac{1}{2}\sin t + \dfrac{1}{4}\sin\sqrt{3}\ t$

e. $\cos\sqrt{3}\ t$

8.   Find the Wronskian determinant of the three functions

$$\{\cos^2 x,\ \sin^2 x,\ 1 \qquad\qquad\} \ .$$

a. $\cos x \sin x$

b. $\tan x$

c. $\sec x$

d.   0

e.   1

PART B (PARTIAL CREDIT)

9.   Use the method of variation of parameters to find a particular solution of the equation $y''' - y' = 1$.

CAUTION: No credit for any other method.

10.   Find the solution of the initial value problem

$$y'' - 4y = \delta(t - 1)$$

$$y(0) = 0,\ y'(0) = 0 \ .$$