**Math 325: Differential Equations** Name:_____
**Exam I** *Sept. 26, 1996* Section:_____

There are 8 problems worth total of 85 points. You start with 15 points.

1. a) (5 points) Show that the functions $\cos^2(x)$, $\cos(2x)$, and 1 are linearly dependent.

b) (5 points) Compute the Wronskian of the functions $e^x$, $e^{2x}$, $e^{3x}$.

2. Consider the differential equation

$$y^{(4)} - y''' + y'' - y' = 0$$

a) (5 points) Find the characteristic polynomial of this equation and determine its roots.

b) (5 points) Find the general solution to this differential equation.

3. (10 points) Using the method of undetermined coefficients, find the form of a particular solution to the differential equation

$$y''' - 3y'' - 9y' + 27y = xe^{3x}$$

Do not solve for the constants.

4. (10 points) Perform two steps of the Euler method in calculating a numerical solution of the differential equation

$$y' = t^2 + ty, \quad y(2) = -1$$

using a step size of $h = 0.25$.

5. a) (5 points) Let $\phi(t)$ be an exact solution of the initial value problem

$$y' = (t+y)^2, \quad y(3) = 4$$

Give the formula for the local truncation error of the Euler method in terms of $\phi(t)$ and the step size $h$.

b) (5 points) Give the formula for computing $(t_{n+1}, y_{n+1})$ from $(t_n, y_n)$ using the Runge-Kutta method for finding a numerical solution to the initial value problem

$$y' = f(t,y), \quad y(t_0) = y_0$$

6. Sketch the graphs and find the Laplace transforms of the functions.

a) (5 points)

$$f(t) = \begin{cases} 1 & 0 \le t \le 1 \\ e^{1-t} & t \ge 1 \end{cases}$$

b) (5 points)

$$f(t) = (t-2)u_2(t) - (t-3)u_3(t)$$

7. Find the inverse Laplace transforms of the functions.

a) (5 points)

$$F(s) = \frac{s}{s^2 + 9} - \frac{1}{s^2 - 4}$$

b) (5 points)

$$F(s) = \frac{e^{-3s}}{s - 1}$$

8. (15 points) Solve the initial value problem using the Laplace transform.

$$y''' - y' = t, \quad y(0) = 0, \quad y'(0) = 2, \quad y''(0) = -1$$