Math 325: Differential Equations Name:____________________________

Quiz 4 *Nov. 8, 1996*

1. The solution to the initial value problem, $\mathbf{X}'(t) = \mathbf{A} \cdot \mathbf{X}(t)$, $\mathbf{X}(0) = \mathbf{X}^{(0)}$, is given by $\mathbf{X}(t) = \exp(\mathbf{A}t) \cdot \mathbf{X}^{(0)}$, where

$$\exp(\mathbf{A}t) = \mathbf{I} + \mathbf{A}t + \mathbf{A}^2\frac{t^2}{2!} + \mathbf{A}^3\frac{t^3}{3!} + \dots$$

Let

$$\mathbf{A} = \begin{bmatrix} 3 & -4 \\ 5 & -6 \end{bmatrix}$$

Calculate the matrix $\exp(\mathbf{A}t)$ another way using the eigenvalues and eigenvectors of $\mathbf{A}$.

2. Suppose a fundamental solution for a homogeneous linear system $\mathbf{X}'(t) = \mathbf{P}(t).\mathbf{X}(t)$ is

$$\mathbf{X}(t) = c_1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} t^2 + c_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} t^3$$

Use the method of variation of parameters to find a particular solution to the non-homogeneous equation

$$\mathbf{X}'(t) = \mathbf{P}(t) \cdot \mathbf{X}(t) + \begin{bmatrix} t \\ -t \end{bmatrix}$$