Math 325: Differential Equations Name:________________________

Exam I *September 23, 1997* Section:________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 10 multiple choice questions worth 5 points each and three partial credit problems worth 10 points each. You start with 20 points.

Determine which set of functions below forms a fundamental set of solutions to the linear differential equation $y''' - \frac{6}{t}y'' = 0$. $1,\ t,\ t^8$ $1,\ t,\ t^3$ $1,\ t,\ e^{6t}$ $1,\ e^{\sqrt{6}\,t},\ e^{-\sqrt{6}\,t}$ $1,\ e^{2t},\ e^{3t}$

Determine the interval in which the solution to the initial value problem

$$(t^2-1)y^{iv} - (t-1)y'' + (t+1)y = t$$
$$y(0) = 1, \quad y'(0) = 0, \quad y''(0) = 1, \quad y'''(0) = 0$$

is guaranteed to exist. $(-1,1)$ $(-1,\infty)$ $(0,\infty)$ $(-\infty,\infty)$ $(-\infty,1)$

Find the general solution of the differential equation $y^{iv} + 4y'' + 4y = 0$. $(c_1t + c_2)\cos(\sqrt{2}t) + (c_3t+c_4)\sin(\sqrt{2}t)$ $(c_1t+c_2)e^{\sqrt{2}t} + (c_3t+c_4)e^{-\sqrt{2}t}$ $c_1\cos(\sqrt{2}t) + c_2\sin(\sqrt{2}t) + (c_3t+c_4)e^{-\sqrt{2}t}$ $c_1\cos(\sqrt{2}t) + c_2\sin(\sqrt{2}t) + c_3e^{\sqrt{2}t} + c_4e^{-\sqrt{2}t}$ $(c_1\cos(\sqrt{2}t) + c_2\sin(\sqrt{2}t)e^{\sqrt{2}t} + (c_3\cos(\sqrt{2}t) + c_4\sin(\sqrt{2}t))e^{-\sqrt{2}t}$

Compute the Wronskian of the functions e^{3t}, $\cos(2t)$, and $\sin(2t)$. $26e^{3t}$ 0 $12e^{3t}(\cos(2t)+\sin(2t))$ $4(\cos(2t)+\sin(2t))$ 36

Compute all the roots of the characteristic polynomial r^3+1. $-1,\ \frac{1}{2}+i\frac{\sqrt{3}}{2},\ \frac{1}{2}-i\frac{\sqrt{3}}{2}$ $-1,\ \frac{\sqrt{3}}{2}+i\frac{1}{2},\ \frac{\sqrt{3}}{2}-i\frac{1}{2}$, $-1,\ 0,\ 1$ $-1,\ i,\ -i$ $-1,\ \frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}$

Let $\phi(t)$ be the solution to the initial value problem $y' = t^4 + \log(y)$, $y(0) = 2$. Determine which of the following expressions gives a correct bound for the local truncation error using the Euler method on the interval $[0,1]$. $|e_{n+1}| \le (2 + \max_{0\le t\le 1}\frac{1+|\log\phi(t)|}{2|\phi(t)|})h^2$ $|e_{n+1}| \le (8 + \frac{1}{2}\max_{0\le t\le 1}|\log\phi(t)|)h^2$ $|e_{n+1}| \le (1+\frac{1}{2}\max_{0\le t\le 1}\log 2)h^2$ $|e_{n+1}| \le (4 + \max_{0\le t\le 1}\frac{1}{2|\phi(t)|})h^2$ $|e_{n+1}| \le \frac{1}{2}(1+\max_{0\le t\le 1}|\phi(t)|)h^2$

Compute the Laplace transform of the function

$$f(t) = \begin{cases} e^t & \text{if } 0 \le t \le 1 \\ 1 & \text{if } t \ge 1 \end{cases}$$

$\frac{1}{1-s}[e^{1-s}-1] + \frac{1}{s}e^{-s}$ $\frac{1}{1-s}[e^{-s}-1] + \frac{1}{s}e^{-s}$ $u_1(s-1)[e^{1-s}-1] + \frac{1}{s}e^{-s}$ $\frac{1}{s}e^{-s}$ $\frac{1}{1-s}[e^{-s}-e^{-1}] + \frac{1}{s}$

Rewrite the following function in terms of step functions.

$$g(t) = \begin{cases} 0 & \text{if } 0 \le t < 2 \\ t-3 & \text{if } 2 \le t < 3 \\ -1 & \text{if } t \ge 3 \end{cases}$$

$(t-3)u_2(t) - (t-2)u_3(t)$ $t - 3 + u_2(t) - (t-3)u_2(t) - u_3(t)$ $(t-3)u_2(t) + (t-3)u_3(t) - u_3(t)$ $u_2(t) + (t-3)u_3(t)$ $(t-3)u_2(t) - u_3(t)$

Compute the inverse Laplace transform of the function

$$F(s) = \frac{1}{s^2-9} - \frac{s}{s^2+25}$$

$\frac{1}{3}\sinh(3t) - \cos(5t)$ $\frac{1}{6}(e^{3t} - e^{-3t}) - \cos(5t)$ $\frac{1}{9}\sinh(3t) - \sin(5t)$ $\frac{1}{9}(e^{9t} - e^{-9t}) - \frac{1}{5}\sin(5t)$ $(e^{3t} - e^{-3t}) - \frac{1}{5}\cos(5t)$

Determine the inverse Laplace transform of the function

$$F(s) = \frac{e^{-2s}}{s-4}$$

$u_2(t)e^{4(t-2)}$ $u_4(t)e^{2(t-4)}$ $u_4(t)e^{4(t-2)}$ $u_2(t)e^{2(t-4)}$ $u_2(t)e^{4(t-4)}$

11. Find the form of the general solution of the equation

$$y''' - 2y'' - 4y' + 8y = te^{2t}$$

(Do not solve for the constants.)

12. *a)* Use the Euler method with step size $h = 0.1$ to calculate an approximate value for $\phi(0.2)$ where $\phi(t)$ is a solution of the initial value problem $y' = t - y^2$, $y(0) = 2$.

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b) Explain how to calculate one step of the Runge-Kutta method to find a numerical approximation to the initial value problem $y' = f(t, y)$, $y(t_0) = y_0$.

13. Use Laplace transforms to solve the initial value problem

$$y'' + 4y = 4t, \qquad y(0) = 0, \qquad y'(0) = 3$$