Math 325: Differential Equations Name:__________________________

Exam II *October 28, 1997* Section:__________________________

0.5trueinRecord your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 10 multiple choice questions worth 5 points each and three partial credit problems worth 10 points each. You start with 20 points.

Solve the initial value problem $y'' + y = u_\pi(t), \quad y(0) = 1, \quad y'(0) = 0.$

$y = \cos(t) + u_\pi(t)[1 - \cos(t-\pi)]$ $y = \sin(t) + u_\pi(t)[1 - \sin(t-\pi)]$ $y = \cos(t) + u_\pi(t)[1 - \sin(t-\pi)]$ $y = \sin(t) + u_\pi(t)[1 - \cos(t-\pi)]$ $y = u_\pi(t)\cos(t) + 1 - \sin(t-\pi)$

Solve the initial value problem $y^{(4)} - y = \delta(t-2)$, $y(0) = 0$, $y'(0) = 0$, $y''(0) = 0$, $y'''(0) = 0$.

$y = u_2(t)\frac{1}{2}[\sinh(t-2) - \sin(t-2)]$ $y = u_2(t)\frac{1}{2}[\cosh(t-2) - \cos(t-2)]$ $y = u_2(t)[\cosh(t-2) + \sin(t-2)]$ $y = \sin(t-2) - u_2(t)\cosh(t-2)$ $y = u_2(t)[\sinh(t) + \sin(t)]$

Determine which of the following expressions is the inverse Laplace transform of

$$F(s) = \frac{1}{s^9(s^2+4)}$$

$\frac{1}{2(8!)}\int_0^t (t-\tau)^8 \sin(2\tau)\,d\tau$ $\frac{1}{2(8!)} t^8 \sin(2t)$ $\frac{1}{2(8!)}\int_0^t (t-\tau)^9 \cos(2\tau)\,d\tau$ $\frac{1}{2(9!)}\int_0^t (t-\tau)^{10} \sin(2\tau)\,d\tau$ $\frac{1}{2(9!)}\int_0^t \tau^9 \sin(2(t-\tau))\,d\tau$

Determine which of the following systems of first order equations corresponds to the differential equation

$$u''' + u'' + 3u' - 5u = \cos(t)$$

$x_1' = x_2$
$x_2' = x_3$
$x_3' = -x_3 - 3x_2 + 5x_1 + \cos(t)$
$x_1' = 2x_2$
$x_2' = x_3$
$x_3' = -x_3 - 3x_2 + 5x_1 + \cos(t)$
$x_1' = x_2$
$x_2' = x_3$
$x_3' = -x_3 - 3x_2 + 5x_1$
$x_1' = x_2$
$x_2' = x_3$
$x_3' = \cos(t)$
$x_1' = x_2$
$x_2' = 2x_3$
$x_3' = -x_3 - 3x_2 + 5x_1 + \cos(t)$

Determine which of the following vectors is linearly dependent on the vectors

$$\mathbf{X}_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \mathbf{X}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

Find an eigenvector corresponding to the eigenvalue -3 for the matrix

$$\begin{bmatrix} -1 & 2 & 2 \\ 2 & 2 & 2 \\ -3 & -6 & -6 \end{bmatrix}$$

$\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

Let $\mathbf{X}_1(t) = \begin{bmatrix} 1 \\ t \end{bmatrix}$ and $\mathbf{X}_2(t) = \begin{bmatrix} t \\ t^2 \end{bmatrix}$. Determine which of the following statements is *not* true. $\mathbf{X}_1(t)$, $\mathbf{X}_2(t)$ are solutions of a system of first order linear equations $\mathbf{X}' = \mathbf{P}(t) \cdot \mathbf{X}$. $W(\mathbf{X}_1(t), \mathbf{X}_2(t)) = 0$ $\mathbf{X}_1(t)$, $\mathbf{X}_2(t)$ are linearly independent vector functions. $\mathbf{X}_1(a)$, $\mathbf{X}_2(a)$ are linearly dependent vectors for any real number a. $\mathbf{X}_2'(t) = t\mathbf{X}_1'(t) + \mathbf{X}_1(t)$

Compute the eigenvalues of the matrix

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 3 \end{bmatrix}$$

0, 2, 4 1, 2, 3 0, 2, 6 0, $2i$, $-2i$ 1, $2i$, $-2i$

Given that $-1 + 2i$ is an eigenvalue and $\begin{bmatrix} -i \\ 1 \end{bmatrix}$ a corresponding eigenvector of a 2×2 matrix $\mathbf{A}$, determine which of the following is a solution to the system $\mathbf{X}' = \mathbf{A} \cdot \mathbf{X}$. $e^{-t} \begin{bmatrix} -\cos(2t) \\ \sin(2t) \end{bmatrix}$ $e^{-t} \begin{bmatrix} -\sin(2t) \\ \cos(2t) \end{bmatrix}$ $e^{-t} \begin{bmatrix} 3\cos(2t) \\ \cos(2t) \end{bmatrix}$ $e^{2t} \begin{bmatrix} \cos(t) \\ \sin(t) \end{bmatrix}$ $e^{2t} \begin{bmatrix} \sin(t) \\ \cos(t) \end{bmatrix}$

Consider the system $\mathbf{X}' = \mathbf{A} \cdot \mathbf{X}$ where $\mathbf{A}$ is the matrix $\begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}$. Determine which type of point the origin is for this system. saddle point node spiral center none of the above

11. Using Laplace transforms and convolution integrals, solve the initial value problem

$$y'' + 4y = g(t), \quad y(0) = 0, \quad y'(0) = 1$$

12. Find the eigenvalues and corresponding eigenvectors of the matrix

$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

13. Find the solution to the initial value problem

$$\begin{aligned} x_1' &= 2x_1 - 6x_2, \quad x_1(0) = 1 \\ x_2' &= -x_1 + x_2, \quad x_2(0) = 0 \end{aligned}$$