Math 325: Differential Equations Name:____________________________

Exam III *November 25, 1997* Section:____________________________

0.5trueinRecord your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 10 multiple choice questions worth 5 points each and three partial credit problems worth 10 points each. You start with 20 points.

Find a fundamental matrix for the system $\mathbf{X}' = \begin{bmatrix} 9 & -8 \\ 12 & -11 \end{bmatrix} \cdot \mathbf{X}$ $\begin{bmatrix} 2e^{-3t} & e^t \\ 3e^{-3t} & e^t \end{bmatrix}$ $\begin{bmatrix} e^{-3t} & e^t \\ e^{-3t} & e^t \end{bmatrix}$ $\begin{bmatrix} 3e^{-3t} & 2e^t \\ e^{-3t} & -e^t \end{bmatrix}$ $\begin{bmatrix} 2e^t & e^{-3t} \\ e^t & 3e^{-3t} \end{bmatrix}$ $\begin{bmatrix} -e^t & e^{-3t} \\ e^t & e^{-3t} \end{bmatrix}$

Suppose

$$\Psi(t) = \begin{bmatrix} 2\cos(t) - \sin(t) & \cos(t) + 2\sin(t) \\ \cos(t) & \sin(t) \end{bmatrix}$$

is a fundamental matrix for a system of linear first order equations. Determine the unique solution through the point $(1, 2)$. $(\cos(t)-8\sin(t), 2\cos(t)-3\sin(t))$ $(\cos(t), 2\cos(t))$ $(\cos(t)-4\sin(t), 2\cos(t)+2\sin(t))$ $(\cos(t)+3\sin(t), 2\cos(t)+8\sin(t))$ $(\cos(t)+2\sin(t), 2\cos(t)-4\sin(t))$

Let $\Psi(t)$ be a fundamental matrix for a homogeneous linear system $\mathbf{X}' = \mathbf{P}(t) \cdot \mathbf{X}$. A solution to the non-homogeneous system $\mathbf{X}' = \mathbf{P}(t) \cdot \mathbf{X} + \mathbf{G}(t)$ can be written in the form $\mathbf{Y}(t) = \Psi(t) \cdot \mathbf{U}(t)$ where $\mathbf{U}(t)$ satisfies: $\mathbf{U}'(t) = \Psi(t)^{-1} \cdot \mathbf{G}(t)$ $\mathbf{U}'(t) = \Psi(t) \cdot \mathbf{G}(t)$ $\mathbf{U}'(t) = \Psi(t) \cdot \Psi(0)^{-1} \cdot \mathbf{G}(t)$ $\mathbf{U}'(t) = \int \Psi(t) \cdot \Psi(0)^{-1} \cdot \mathbf{G}(t)$ $\mathbf{U}'(t) = \int \Psi(t)^{-1} \cdot \mathbf{G}(t)$

Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Which of the following matrices equals $\exp(At)$. $\begin{bmatrix} \cosh(t) & \sinh(t) \\ \sinh(t) & \cosh(t) \end{bmatrix}$ $\begin{bmatrix} \cos(t) & -\sin(t) \\ -\sin(t) & \cos(t) \end{bmatrix}$ $\begin{bmatrix} e^t & e^{-t} \\ e^t & -e^{-t} \end{bmatrix}$ $\frac{1}{2}\begin{bmatrix} -e^t & e^{-t} \\ e^t & e^{-t} \end{bmatrix}$ $\begin{bmatrix} e^t & 0 \\ 0 & e^{-t} \end{bmatrix}$

Determine the type and stability of the critical point $(0, 0)$ for the system

$$\frac{dx}{dt} = 2x + 2y$$
$$\frac{dy}{dt} = 2x - 2y$$

saddle unstable node unstable spiral stable spiral stable node

Consider the non-linear system

$$\frac{dx}{dt} = \cos(x) - y^2$$
$$\frac{dy}{dt} = e^{xy} - 1$$

Find the coefficient matrix for the corresponding linear system at the critical point $(0, 1)$. $\begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix}$ $\begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}$ $\begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} -3 & 0 \\ 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$

Determine the critical points of the system

$$\frac{dx}{dt} = x - x^2 - xy$$
$$\frac{dy}{dt} = 2y - y^2 - 3xy$$

$(0,0),\ (0,2),\ (1,0),\ (1/2,1/2)$ $(0,0),\ (0,2),\ (1,0),\ (1,1)$ $(0,0),\ (0,1),\ (2,0),\ (2,2)$ $(0,0),\ (0,1/2),\ (1,0),\ (1/2,1/2)$ $(0,0),\ (2,0),\ (0,1),\ (1,1)$

Determine the type and stability of the critical point $(0,0)$ for the system

$$\frac{dx}{dt} = 2x + 4y^2$$
$$\frac{dy}{dt} = x + 2y$$

unstable node or spiral asymptotically stable node center, stability indeterminate unstable saddle unstable spiral

Identify which type of system most closely matches the following phase portrait.

predator-prey system undamped pendulum damped pendulum competing species with stable coexistence competing species with extinction of one species

The following system models populations of competing species.

$$\frac{dx}{dt} = x(2 - 2x - y)$$
$$\frac{dy}{dt} = y(4 - 2x - 3y)$$

Deteremine the limiting populations $x(t)$, $y(t)$, as $t \to \infty$, if the starting populations are $x(0) = 3$ and $y(0) = 4$. $x(t) \to 1/2,\ y(t) \to 1$ $x(t) \to 1,\ y(t) \to 0$ $x(t) \to 0,\ y(t) \to 4/3$ $x(t) \to 0,\ y(t) \to 0$ $x(t) \to 3/2,\ y(t) \to 2$

11. Find the general solution of the following system using the method of variation of parameters.

$$\frac{dx}{dt} = -10x + 3y - 3e^{-t}$$
$$\frac{dy}{dt} = -36x + 11y - 6e^{-t}$$

12. Determine all the real critical points of the following system and discuss the type and stability of each.

$$\frac{dx}{dt} = 1 - xy$$
$$\frac{dy}{dt} = x - y^3$$

13. Consider the following system that models two interacting species with populations x and y.

$$\frac{dx}{dt} = x(1 - \frac{1}{2}y)$$
$$\frac{dy}{dt} = y(-\frac{1}{3} + \frac{2}{3}x)$$

(a) Find the critical points of this system and describe their stability properties.
(b) Sketch a phase portrait for this system.