Math 325: Differential Equations Name:____________________________

Final Exam *December 19, 1997* Section:____________________________

0.5trueinRecord your answers by placing an $\times$ through one letter for each problem on this answer sheet. There are 20 multiple choice questions worth 6 points each. You start with 30 points.

Find the general solution of $y^{(4)} + y^{(2)} = 0$.

$y = c_1+c_2t+c_3\cos(t)+c_4\sin(t)$ $y = c_1t+c_2t^2+c_3\cos(t)+c_4\sin(t)$ $y = (c_1+c_2t)\cos(t)+(c_3+c_4t)\sin(t)$ $y = (c_1t+c_2t^2)\cos(t)+(c_3t+c_4t^2)\sin(t)$ $y = c_1t+c_2t^2+c_3e^t+c_4e^{-t}$

Determine the form of a particular solution to the non-homogeneous equation

$$y''' - 4y'' + 4y' = t + e^{2t}$$

$y = At^2+Bt+Ct^2e^{2t}$ $y = At^3+Bt^2+Cte^{2t}$ $y = At^2+Bt+Cte^{2t}$ $y = At^3+Bt^2+Ct^2e^{2t}$ $y = At^3+Bt^2+(Ct^2+Dt)e^{2t}$

Given that $\mathbf{X} = c_1\begin{bmatrix} 2 \\ -1 \end{bmatrix} + c_2\begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{5t}$ is general solution of the homogeneous system $\mathbf{X}' = \mathbf{A}\cdot\mathbf{X}$, use the method of variation of parameters to find a particular solution of the non-homogeneous system $\mathbf{X}' = \mathbf{A}\cdot\mathbf{X} + \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^t$.

$\mathbf{X} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^t$ $\mathbf{X} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $\mathbf{X} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^t$ $\mathbf{X} = \begin{bmatrix} 2e^{5t} \\ -e^t \end{bmatrix}$ $\mathbf{X} = \begin{bmatrix} e^t \\ 2e^{5t} \end{bmatrix}$

Find the fundamental matrix $\Phi(t)$ with $\Phi(0) = \mathbf{I}$ for the system

$$\mathbf{X}' = \begin{bmatrix} 3 & 6 \\ -1 & -2 \end{bmatrix}\cdot\mathbf{X}$$

$\begin{bmatrix} 3e^t-2 & 6e^t-6 \\ 1-e^t & 3-2e^t \end{bmatrix}$ $\begin{bmatrix} 1 & 3-3e^t \\ t & e^t \end{bmatrix}$ $\begin{bmatrix} \cos(t) & \sin(2t) \\ -\sin(t) & \cos(2t) \end{bmatrix}$ $\begin{bmatrix} 3e^t-2 & 6e^t-6 \\ e^t-1 & e^t \end{bmatrix}$ $\begin{bmatrix} \cos(3t) & \sin(t) \\ \sin(3t) & \cos(t) \end{bmatrix}$

Determine which of the following vectors is an eigenvector of the matrix

$$\begin{bmatrix} 0 & 0 & -1 \\ 2 & 0 & 0 \\ -1 & 2 & 4 \end{bmatrix}$$

corresponding to the eigenvalue $r = 1$.

$\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ $\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

Solve the initial value problem

$$\mathbf{X}' = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}\cdot\mathbf{X}, \quad \mathbf{X}(0) = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$\begin{bmatrix} 3\cos(2t)+4\sin(2t) \\ -3\sin(2t)+4\cos(2t) \end{bmatrix}$ $\begin{bmatrix} 3\cos(2t) \\ 4\cos(2t) \end{bmatrix}$ $\begin{bmatrix} 3 \\ 4 \end{bmatrix} e^{2t}$ $\begin{bmatrix} 3\cosh(2t) \\ 4\cosh(2t) \end{bmatrix}$ $\begin{bmatrix} 4\cos(2t)-3\sin(2t) \\ 3\cos(2t)+4\sin(2t) \end{bmatrix}$

Find the critical points of the system

$$\frac{dx}{dt} = x^2 + xy$$
$$\frac{dy}{dt} = y^3 + x^2$$

$(0,0)$, $(1,-1)$ $(0,0)$, $(1,-1)$, $(-1,1)$ $(0,0)$, $(1,1)$, $(-1,-1)$ $(0,0)$, $(1,-1)$, $(1,1)$, $(-1,1)$, $(-1,-1)$ $(0,0)$, $(-1,1)$

Determine the type and stability of the critical point $(1,1)$ for the system

$$\frac{dx}{dt} = 2x - 3x^2y + x^3$$
$$\frac{dy}{dt} = y + xy^2 - 2y^3$$

stable spiral stable node unstable node unstable saddle point unstable spiral

The following nonlinear system models populations of competing species:

$$\frac{dx}{dt} = x(2 - y - x)$$
$$\frac{dy}{dt} = y(3 - 2x - y)$$

Determine which phase portrait corresponds to this system.

Find the Fourier series for the function $f(x) = \begin{cases} 1 - x & 0 \leq x \leq 1 \\ -1 - x & -1 \leq x \leq 0 \end{cases}$

$f(x) = \frac{2}{\pi}\sin(\pi x) + \frac{2}{2\pi}\sin(2\pi x) + \frac{2}{3\pi}\sin(3\pi x) + \frac{2}{4\pi}\sin(4\pi x) + \cdots$ $f(x) = \frac{3}{\pi}\sin(\pi x) + \frac{1}{2\pi}\sin(2\pi x) + \frac{3}{3\pi}\sin(3\pi x) + \frac{1}{4\pi}\sin(4\pi x) + \cdots$ $f(x) = \frac{2}{\pi}\sin(\pi x) - \frac{1}{2\pi}\sin(2\pi x) + \frac{2}{3\pi}\sin(3\pi x) - \frac{1}{4\pi}\sin(4\pi x) + \cdots$ $f(x) = \frac{2}{\pi}\sin(\pi x) - \frac{2}{3\pi}\sin(3\pi x) + \frac{2}{5\pi}\sin(5\pi x) - \frac{2}{7\pi}\sin(7\pi x) + \cdots$ $f(x) = \frac{1}{\pi}\sin(\pi x) + \frac{2}{3\pi}\sin(3\pi x) + \frac{1}{5\pi}\sin(5\pi x) + \frac{2}{7\pi}\sin(7\pi x) + \cdots$

Use the method of separation of variables to replace the partial differential equation

$$(xu_x)_x + u_{tt} = 0$$

by a pair of ordinary differential equations.

$\begin{cases} xX''(x) + X'(x) - \sigma X(x) = 0 \\ T''(t) + \sigma T(t) = 0 \end{cases}$ $\begin{cases} X''(x) - \sigma X(x) = 0 \\ T''(t) - \sigma T(t) = 0 \end{cases}$ $\begin{cases} X''(x) + xX'(x) - \sigma X(x) = 0 \\ T''(t) + \sigma = 0 \end{cases}$

$\begin{cases} X''(x) - \sigma x X'(x) = 0 \\ T''(t) + \sigma t T(t) = 0 \end{cases}$ $\begin{cases} X''(x) + \sigma x X'(x) = 0 \\ T''(t) - \sigma T(t) = 0 \end{cases}$

Solve the heat equation $5u_{xx} = u_t$ for $0 \leq x \leq 10$, $t \geq 0$, subject to the boundary conditions $u(0,t) = 0$, $u(10,t) = 0$ for $t \geq 0$ and the initial condition $u(x,0) = 30$ for $0 \leq x \leq 10$.

$u(x,t) = \frac{120}{\pi}\sum_{k=1}^{\infty} \frac{1}{2k-1}\sin(\frac{(2k-1)\pi}{10}x)e^{-((2k-1)\pi/10)^2 5t}$ $u(x,t) = \frac{60}{\pi}\sum_{k=1}^{\infty} \frac{1}{2k}\sin(\frac{2k\pi}{10}x)e^{-(2k\pi/10)^2 5t}$ $u(x,t) = \frac{120}{\pi}\sum_{k=1}^{\infty} \frac{1}{k}\sin(\frac{k\pi}{10}x)e^{-(k\pi/10)^2\sqrt{5}t}$ $u(x,t) = \frac{60}{\pi}\sum_{k=1}^{\infty} \frac{(-1)^k}{k}\sin(\frac{k\pi}{10}x)e^{-(k\pi/10)^2\sqrt{5}t}$ $u(x,t) = \frac{120}{\pi}\sum_{k=1}^{\infty} \frac{(-1)^k}{2k-1}\sin(\frac{k\pi}{10}x)e^{-(k\pi/10)^2 t}$

Solve the wave equation $9u_{xx} = u_{tt}$ for $0 \le x \le \pi$, $t \ge 0$, subject to the boundary conditions $u(0,t) = 0$, $u(\pi,t) = 0$ for $t \ge 0$ and the initial conditions $u(x,0) = -x$, $u_t(x,0) = 0$ for $0 \le x \le \pi$.

$u(x,t) = 2\sum_{n=1}^{\infty} \frac{(-1)^n}{n}\sin(nx)\cos(3nt)$ $u(x,t) = 2\sum_{n=1}^{\infty} \frac{(-1)^n}{n}\sin(nx)\cos(nt)$ $u(x,t) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n\pi}\sin(\frac{n\pi}{3}x)\cos(\frac{n}{3}t)$ $u(x,t) = \sum_{n=1}^{\infty} \frac{1}{n\pi}\sin(\frac{n\pi}{3}x)\cos(\frac{n\pi}{3}t)$ $u(x,t) = \frac{2}{3\pi}\sum_{n=1}^{\infty} \frac{1}{n^2}\sin(nx)\cos(3nt)$

Solve the heat equation $u_{xx} = u_t$, $0 \le x \le 3$, $t \ge 0$, subject to the boundary conditions $u(0,t) = 0$, $u(3,t) = 3$ for $t \ge 0$ and the initial condition $u(x,0) = x$ for $0 \le x \le 3$.

$u(x,t) = x$ $u(x,t) = x + \frac{6}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^n}{n}\sin(\frac{n\pi}{3}x)e^{-(n\pi/3)^2 t}$ $u(x,t) = -\frac{6}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^n}{n}\sin(\frac{n\pi}{3}x)e^{-(n\pi/3)}$ $u(x,t) = \frac{6}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^n}{n}\sin(\frac{n\pi}{3}x)\cos(\frac{n\pi}{3}t)$ $u(x,t) = \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^n}{n}\sin(\frac{n\pi}{3}x)\sin(\frac{n\pi}{3}t)$

Find the inverse Laplace transform of the function $\frac{4}{(s-1)^3}$.

$2t^2e^t$ $2t^2$ $4t^2e^{4t}$ te^t t^3e^t

Find the inverse Laplace transform of $\frac{1-e^{-2s}}{s^2}$.

$t - u_2(t)(t-2)$ $t^2 - u_2(t)(t-2)$ $t - u_2(t)t^2$ $t - u_2(t)t$ $t^2 - u_2(t)\sin(t-2)$

Determine the solution of the initial value problem

$$y'' + 2y' + 2y = \delta(t-\pi), \quad y(0) = 1, \quad y'(0) = 0$$

$y(t) = e^{-t}(\cos(t) + \sin(t)) + u_\pi(t)e^{-t+\pi}\sin(t-\pi)$ $y(t) = e^{-2t}(\cos(2t) + \sin(2t)) + u_\pi(t)e^{-t+\pi}\sin(t-\pi)$ $y(t) = 2u_\pi(t)e^{-t+\pi}\sin(t)$ $y(t) = e^{-t}(\cos(t)+\sin(t))$ $y(t) = e^{-t}(\cos(t)+\sin(t)) + u_\pi(t)\sin(t-\pi)$

Find the general solution of the system

$$\mathbf{X}' = \begin{bmatrix} 4 & -3 \\ 8 & -6 \end{bmatrix} \cdot \mathbf{X}$$

$\mathbf{X} = c_1\begin{bmatrix} 3 \\ 4 \end{bmatrix} + c_2\begin{bmatrix} 1 \\ 2 \end{bmatrix}e^{-2t}$ $\mathbf{X} = c_1\begin{bmatrix} 3 \\ 4 \end{bmatrix}te^{-2t} + c_2\begin{bmatrix} 1 \\ 2 \end{bmatrix}e^{-2t}$ $\mathbf{X} = c_1\begin{bmatrix} 4 \\ -3 \end{bmatrix}e^{-2t} + c_2\begin{bmatrix} 2 \\ -1 \end{bmatrix}e^{-2t}$ $\mathbf{X} = c_1\begin{bmatrix} 3 \\ 4 \end{bmatrix}e^{t} + c_2\begin{bmatrix} 1 \\ 2 \end{bmatrix}e^{-t}$ $\mathbf{X} = c_1\begin{bmatrix} 4 \\ -3 \end{bmatrix} + c_2\begin{bmatrix} 2 \\ -1 \end{bmatrix}e^{-2t}$

Compute the Wronskian of

$$\mathbf{X}_1 = \begin{bmatrix} \cos(t) \\ \sin(t) \end{bmatrix}e^t, \quad \mathbf{X}_2 = \begin{bmatrix} \sin(t) \\ \cos(t) \end{bmatrix}e^t$$

$e^{2t}(\cos^2(t) - \sin^2(t))$ $e^t(\cos^2(t) - \sin^2(t))$ e^{2t} e^t 0

Use the Euler method with step size $h = 0.5$ to find an approximation of $\phi(1)$ where $\phi(t)$ is the solution to the initial value problem

$$y' = t^2 + y^2, \quad y(0) = 1$$

2.75 1.25 1.50 4.0 2.25