Math 325: Differential Equations Name:____________________

Exam II *October 27, 1998* Section:____________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 8 multiple choice questions worth 9 points each and two partial credit problems worth 14 points each.

Given $\mathbf{x}_1 = (1, -1)$. Which $\mathbf{x}_2$ below will make $\mathbf{x}_1, \mathbf{x}_2$ linearly independent. $\mathbf{x}_2 = (2, 1)$ $\mathbf{x}_2 = (-1, 1)$ $\mathbf{x}_2 = (2, -2)$ $\mathbf{x}_2 = (0, 0)$ $\mathbf{x}_2 = (-4, 4)$

Compute the eigenvalues of the matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 3 & -1 \\ -1 & 2 & -1 \end{pmatrix}$$

$1, 1+\sqrt{2}, 1-\sqrt{2}$ $1, 1+\sqrt{2}i, 1-\sqrt{2}i$ $1, \sqrt{2}i, -\sqrt{2}i$ $0,\ 12(1+\sqrt{2}),\ 12(1-\sqrt{2})$ $1,\ -1,\ -2$

Under the change of variables $x_1 = y$, $x_2 = y'$ and $x_3 = y''$, which of the following systems corresponds to the equation $y''' + 4y'' + 5y' + 6y = 7\sin t + 8\cos 2t$?

$$\begin{cases} x_1' = x_2 \\ x_2' = x_3 \\ x_3' = -4x_3 - 5x_2 - 6x_1 + 7\sin t + 8\cos 2t \end{cases}$$

$$\begin{cases} x_1' = x_2 \\ x_2' = x_3 \\ x_3' = -4x_1 - 5x_2 - 6x_3 + 7\sin t + 8\cos 2t \end{cases}$$

$$\begin{cases} x_1' = x_2 \\ x_2' = x_3 \\ x_3' = 4x_1 + 5x_2 + 6x_3 + 7\sin t + 8\cos 2t \end{cases}$$

$$\begin{cases} x_1' = x_2 \\ x_2' = x_3 \\ x_3' = 4x_1 + 5x_2 + 6x_3 - 7\sin t - 8\cos 2t \end{cases}$$

$$\begin{cases} x_1' = x_2 \\ x_2' = x_3 \\ x_3' = 4x_3 + 5x_2 + 6x_1 + 7\sin t + 8\cos 2t \end{cases}$$

Find all linearly independent eigenvector(s) corresponding to the eigenvalue $r = 1$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

Given that $\mathbf{A}$ is a real 2x2 matrix. -1, 2 are the eigenvalues and $\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ are the corresponding eigenvectors. Then the general solution for $\mathbf{x}' = \mathbf{A}\cdot\mathbf{x}$ is given by

$c_1 e^{-t}\begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 e^{2t}\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ $c_1 e^{2t}\begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 e^{-t}\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ $c_1 e^{-t}\begin{pmatrix} \cos t \\ 2\sin t \end{pmatrix} + c_2 e^{2t}\begin{pmatrix} -2\sin t \\ \cos t \end{pmatrix}$ $c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 e^{-t}\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ $c_1 e^{2t}\begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

Given that $\mathbf{A}$ is a real 2x2 matrix. $2-i$ is an eigenvalue and $\begin{pmatrix} 3 \\ i \end{pmatrix}$ is an eigenvector. Determine which of the following is a general solution to the first order ODE system $\mathbf{x}' = \mathbf{A} \cdot \mathbf{x}$ $c_1 e^{2t} \begin{pmatrix} 3\cos t \\ \sin t \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} -3\sin t \\ \cos t \end{pmatrix}$ $c_1 e^{2t} \begin{pmatrix} -\cos t \\ 3\sin t \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 3\sin t \\ -\cos t \end{pmatrix}$ $c_1 e^{2t} \begin{pmatrix} \cos t \\ -3\sin t \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} -\sin t \\ 3\cos t \end{pmatrix}$ $c_1 e^{-t} \begin{pmatrix} \cos 2t \\ -3\sin 2t \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} -\sin t \\ 3\cos t \end{pmatrix}$ $c_1 e^{-t} \begin{pmatrix} 3\cos 2t \\ \sin 2t \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} 3\sin 2t \\ -\cos 2t \end{pmatrix}$

Given that $\mathbf{A}$ is a real 3x3 matrix. -4 is an double eigenvalue and $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$ are corresponding eigenvectors. A third eigenvalue is 5 and the corresponding eigenvector is $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ Then the general solution for $\mathbf{x}' = \mathbf{A} \cdot \mathbf{x}$ is given by $c_1 e^{-4t} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + c_2 e^{-4t} \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + c_4 e^{5t} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $c_1 e^{-4t} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + c_2 t e^{-4t} \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + c_4 e^{5t} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $c_1 t e^{-4t} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + c_2 e^{-4t} \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + c_4 e^{5t} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $c_1 (e^{-4t} + t e^{-4t}) \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + c_2 (e^{-4t} + t e^{-4t}) \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + c_4 e^{5t} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ The general solution does not exist.

Determine which of the following statements is NOT true. If $\mathbf{A}$ is an 3 by 3 real matrix, then all the eigenvalues are real.

If $\mathbf{A}$ is an 3 by 3 real and symmetric matrix, then there always exist 3 linearly independent eigenvectors, even if some of the eigenvalues are repeated.

Let $\mathbf{A}$ be an 3 by 3 real matrix. If $\det \mathbf{A} = 0$, then the column vectors of $\mathbf{A}$ are linearly dependent.

Let $\mathbf{x}^{(1)}(t)$ and $\mathbf{x}^{(2)}(t)$ be two solutions of $\mathbf{x}' = \mathbf{A}\mathbf{x}$, where $\mathbf{A}$ is a 2x2 matrix. If $\mathbf{x}^{(1)}(t)$, $\mathbf{x}^{(2)}(t)$ are linearly independent at $t = 0$, then they are also linearly independent at $t = 1$.

If ξ is an eigenvector corresponding to a repeated eigenvalue $r = \rho$, then $e^{\rho t}\xi$ is a solution to the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$.

(a). Find the general solution of the homogeneous system

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} \mathbf{x}$$

(b). Find a particular solution of the non-homogeneous system

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 \\ e^{2t} \end{pmatrix}$$

Consider the system

$$\begin{cases} x_1' = -5x_1 + 3x_2 \\ x_2' = -3x_1 + x_2 \end{cases}$$

(a). Find the eigenvalues and eigenvectors.

(b). Find the general solution.