February 18, 1998

Ordinary Differential Equations, MATH 325, Exam 1

**Name:**

This test consists of 5 partial credit problems. It will be exactly 50 min in length. When you are told to begin, but not before, glance through the entire test and put your name on each page. It is YOUR RESPONSIBILITY to make sure that your test consists of 6 PAGES with 5 PROBLEMS. The total point value of the test is 100 points. Use the back of the test pages for scratch work.

**I have neither given nor received unauthorized aid on this exam:**

1. (20 points) Find the general solution to

a) (10)

$$y^{(4)} - y^{(3)} - 3y'' + 5y' - 2y = 0$$

Hint: The auxiliary equation is: $r^4 - r^3 - 3r^2 + 5r - 2 = (r-1)^3(r+2) = 0$

b) (10)

$$y^{(4)} - 8y^{(3)} + 26y'' - 40y' + 25y = 0$$

Hint: The auxiliary equation is: $r^4 - 8r^3 + 26r^2 - 40r + 25 = (r^2 - 4r + 5)^2 = 0$

2. (25 points) Solve the following initial value problem:

$y^{(3)} - 4y'' + 7y' - 6y = 0$
$y(0) = 1$
$y'(0) = 0$
$y''(0) = 0$

Hint: The auxiliary equation of the homogeneous differential equation is:
$r^3 - 4r^2 + 7r - 6 = (r - 2)(r^2 - 2r + 3) = 0$

3. (15 points) Use the method of undetermined coefficients to find a particular solution to:

$$y^{(3)} - 3y'' + 4y' = xe^{2x}$$

Hint: The auxiliary equation is: $r^3 - 3r^2 + 4 = (r+1)(r-2)^2 = 0$

4. (25 points)

a) (10) Calculate the Laplace transform of the following function:

$$f(x) = \begin{cases} 1, & 0 < t < 2 \\ 0, & 2 < t < 5 \\ e^{3t}, & 5 < t \end{cases}$$

b) (15) Solve the following initial value problem using the method of Laplace transforms:

$$y'' + y = t^2 + 2, \quad y(0) = 1, \quad y'(0) = -1.$$

5. (15 points)

a) (5) Why is improved Euler's method better then Euler's method? Justify your answer.

b) (10) Give the formula for computing $(t_{n+1}, y_{n+1})$ in terms of $(t_n, y_n)$ using the 4-th order Runge-Kutta method for finding a numerical solution to the initial value problem

$$y' = t^2 + y^2, \quad y(t_0) = 0.$$