

March 20, 1998

Ordinary Differential Equations, MATH 325, Exam 2

Name:

This test consists of 4 partial credit problems. It will be exactly 50 min in length. When you are told to begin, but not before, glance through the entire test and put your name on each page. It is YOUR RESPONSIBILITY to make sure that your test consists of 5 PAGES with 4 PROBLEMS. The total point value of the test is 100 points. Use the back of the test pages for scratch work.

I have neither given nor received unauthorized aid on this exam:

1. (30 points) Use the Laplace transform to solve the initial value problem

$$y'' + 2y = u_{2\pi}(t) - u_{4\pi}(t), \quad y(0) = y'(0) = 1$$

where $u_{2\pi}(t)$ and $u_{4\pi}(t)$ are step functions.

2. (10 points) Use the convolution integral to find inverse Laplace transforms of the following functions.

a) $\frac{1}{s(s^2 + 1)}$

b) $\frac{s}{(s^2 + 1)^2}$

3. (30 points)

a) (25) Use the Laplace transform to solve the initial value problem

$$y'' + y = \delta(t - 2\pi), \quad y(0) = 0, y'(0) = 1$$

where $\delta(t)$ is the Dirac delta function.

b) (5) Sketch a graph of the solution.

4. (30 points) Transform the following system into a single differential equation of second order:

$$\begin{aligned}x_1' &= 2x_1 - x_2 \\x_2' &= -x_1 + 2x_2.\end{aligned}$$

Then find x_1 and x_2 that also satisfy the given initial conditions: $x_1(0) = -1, x_2(0) = -2$.