April 22, 1998

Ordinary Differential Equations, MATH 325, Exam 3

Name:

This test consists of 5 partial credit problems. It will be exactly 50 min in length. When you are told to begin, but not before, glance through the entire test and put your name on each page. It is YOUR RESPONSIBILITY to make sure that your test consists of 6 PAGES with 5 PROBLEMS. The total point value of the test is 100 points. Use the back of the test pages for scratch work.

I have neither given nor received unauthorized aid on this exam:

1. (15 points) Suppose that the given vectors X_1 and X_2 are solutions of a system of linear differential equations: $\dfrac{dX}{dt} = AX$. Determine whether the vectors form a fundamental set on $(-\infty, \infty)$

$$X_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t, \quad X_1 = \begin{pmatrix} 2 \\ 6 \end{pmatrix} e^t + \begin{pmatrix} 8 \\ -8 \end{pmatrix} te^t.$$

(2) (20 points) Find the general solution of the following system:

$$X' = \begin{pmatrix} 3 & -18 \\ 2 & -9 \end{pmatrix} X.$$

(3) (20 points) The following system has a single critical point at (x_0, y_0):

$$
\begin{aligned}
x' &= x - 2y \\
y' &= 3x - 4y - 2.
\end{aligned}
$$

Classify it as to type and stability.

Hint: Begin by finding (x_0, y_0) and by making substitution of the form:
$u = x - x_0, v = y - y_0$.

(4) (20 points) Investigate the type and stability of the critical point (0,0) of the following system. Begin by showing that it is almost linear.

$$x' = x + 2y + x^2 + y^2$$
$$y' = 2x - 2y - 3xy.$$

(5) (25 points) Find all critical points of the following almost linear system and investigate the type and stability of each point:

$$\begin{aligned} x' &= x - y \\ y' &= x^2 - y. \end{aligned}$$

Hint: Assume that the system is almost linear. Do NOT show that it is almost linear.