May 5, 1998

Ordinary Differential Equations, MATH 325, Final Exam

Name:

This test consists of 10 partial credit problems. It will be exactly 2 hours in length. When you are told to begin, but not before, glance through the entire test and put your name on each page. It is YOUR RESPONSIBILITY to make sure that your test consists of 11 PAGES with 10 PROBLEMS. The total point value of the test is 150 points. Use the back of the test pages for scratch work.

I have neither given nor received unauthorized aid on this exam:

1. (10 points) Find the general solution for the differential equation

$$y^{(3)} + 3y^{(2)} + 28y' + 26y = 0.$$

Hint: One of the roots of the auxiliary equation is: $r = -1$.

2. (20 points) Solve the following initial value problem:

$y^{(3)} - y'' - 4y' + 4y = 0$
$y(0) = -4$
$y'(0) = -1$
$y''(0) = -19$

Hint: One of the roots of the auxiliary equation is: $r = 1$.

3. (10 points) Calculate the Laplace transform of the each of the following functions:

a)

$$f_1(x) = \begin{cases} 0, & t = 2 \\ t, & t \neq 2 \end{cases}$$

b)

$$f_2(x) = \begin{cases} 5, & t = 1 \\ 2, & t = 6 \\ t, & t \neq 1, 6 \end{cases}$$

c)

$$f_3(x) = t.$$

d) Which of the preceding functions is the inverse Laplace transform of $\dfrac{1}{s^2}$?

Justify your answer.

4. (15 points) Express the given function using unit step functions and compute its Laplace transform

$$g(t) = \begin{cases} 0, & t < 1 \\ 2, & 1 < t < 2 \\ 1, & 2 < t < 3 \\ 3, & 3 < t. \end{cases}$$

5. (15 points) Solve the following initial value problem using the method of Laplace transforms:

$$y'' + 5y' - 6y = 21e^t, \quad y(0) = -1, \quad y'(0) = 9.$$

6. (15 points) Solve the following initial value problem using the method of Laplace transforms:

$$y'' + y' = u_3(t), \quad y(0) = 0, \quad y'(0) = 1.$$

7. (10 points) Use the convolution integral to find inverse Laplace transforms of the following function:

$$Y(s) = \frac{s}{(s-1)(s+2)}.$$

Hint: $\dfrac{s}{(s-1)} = 1 + \dfrac{1}{(s-1)}$

8. (20 points) Use the Laplace transform to solve the initial value problem

$$y'' + 2y' - 3y = \delta(t-1) - \delta(t-2), \quad y(0) = 2, \ \ y'(0) = -2$$

where $\delta(t)$ is the Dirac delta function.

9. (15 points)

a) (5) Explain the method of separation of variables in case of a Heat equation.

b) (10) Consider $\tilde{f}(x)$, odd periodic extension of period 2π of the function:

$$f(x) = 2, \quad 0 < x < \pi.$$

1. Sketch the graph of the function $\tilde{f}(x)$ and calculate its Fourier series.
2. What is $\tilde{f}(\pi) =?$

10 (20 points)

a) (10) Show that the following system is almost linear:

$$\begin{aligned} x' &= -2xy \\ y' &= y - x + xy - y^3. \end{aligned}$$

b) (10) Find all critical points of this system and investigate the type of the critical point $(0, -1)$.