Name: ______________________ **Instructor:** ____________

Mathematics 325
Fall Semester 2000
Test 1, Version A
September 14, 2000

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1. This Examination contains twelve problems worth a total of 100 points. The test will be exactly 75 minutes in length.

2. For each multiple choice question, please mark an X on the correct answer on the answer sheet. **Do not circle it.**

3. On problem 10, 11 and 12, show your work, indicating clearly in the space provided how you arrive at your answer; otherwise no credit will be given.

4. Calculators, books and notes are not allowed.

5. A table of Laplace transforms is supplied at the end of the booklet.

6. Hand in the entire test.

Sign the pledge: "On my honor, I have neither given nor received unauthorized aid on this Exam":

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GOOD LUCK

PART A

1. Find the general solution of

$$y''' - y'' - y' + y = 0.$$

a. $C_1e^t + C_2e^{-t} + C_3te^{2t}$.

b. $C_1e^t + C_2\sin t + C_3\cos t$.

c. $C_1te^t + C_2e^t + C_3e^{-t}$.

d. $C_1te^t + C_2\sin t + C_3\cos t$.

e. $C_1e^t + C_2te^{-t} + C_3e^{-t}$.

2. Which of the following is a solution of the equation

$$y''' + 8y'' + 25y' = 0.$$

a. $e^{3t}\cos 5t$.

b. $e^{4t}\sin 3t$.

c. $e^{-4t}\cos 3t$.

d. $e^{-3t}\cos 4t$.

e. $e^{-5t}\cos 4t$.

3. Find the form of a particular solution of

$$y''' - 3y'' + 3y' - y = e^t.$$

(Hint, 1 is a triple root of the characteristic polynomial $Z(r) = (r-1)^3$.)

a. Ce^t.

b. Cte^t.

c. $e^{-t}(A_0t^3 + A_1t^2 + A_2t + A_3)$.

d. $A_0t^3e^t$.

e. $C(A_0t + A_1)$.

4. Find the Wronskian of the functions $\{e^x, e^{2x}, e^{3x}\}$.

   a. e^{6x}

   b. $3e^{6x}$

   c. $6e^{6x}$.

   d. 0

   e. $2e^{6x}$

5. Let $f(t)$ be the function defined by

$$f(t) = \begin{cases} t-2, & \text{if } t \le 2 \\ 0, & \text{if } t > 2, \end{cases}$$

   Find the Laplace transform $F(s)$ of f.

   a. $\dfrac{1}{s^2} - \dfrac{1}{s} - \dfrac{e^{-2s}}{s}$.

   b. $\dfrac{1}{s^2} - \dfrac{2}{s} - \dfrac{e^{-2s}}{s^2}$.

   c. $\dfrac{2}{s^2} - \dfrac{1}{s} - \dfrac{e^{-s}}{s}$.

   d. $\dfrac{1}{s} - \dfrac{e^{-2s}}{s^2}$.

   e. $\dfrac{1}{s^2}(1 - e^{-2s})$.

6. Find the inverse Laplace transform of $F(s) = \dfrac{4}{s^2 - 6s + 10}$

   a. $4e^{3t}\sin t$. b. $3e^{4t}\sin t$. c. $4e^{3t}\cos 3t$. d. $4e^{3t}\cos t$. e. $4e^{3t}t$.

7. Find the Laplace transform of the function $f(t) = \delta(t-1)(t^{99} + 99)$.

a. $99e^{-s}$.

b. $100e^{-99s}$.

c. $99e^{s}$.

d. $s^{-99}e^{-s}$.

e. $100e^{-s}$.

8. Find the inverse Laplace transform of $F(s) = \dfrac{se^{-2s}}{s^2 - 9}$. (Hint: use the a table of Laplace transforms supplied at at the end of the booklet.)

a. $u_2(t)\cosh 3(t-2)$.

b. $u_2(t)\sinh 3(t-2)$.

c. $u_3(t)\sin 2(t-2)$.

d. $u_9(t)\sinh 2(t-1)$.

e. $u_3(t)\cos 2(t-3)$.

9. Find the solution to the following initial value problem

$$y''' + 8y'' + 8y' + 9y = 1; \quad y(0) = 1,\ y'(0) = y''(0) = 0.$$

a. $y = \frac{1}{9}$.

b. $y = \mathcal{L}^{-1}(\dfrac{s^{-1} + s^2 + 8s + 8}{s^3 + 8s^2 + 8s + 9})$.

c. $y = \mathcal{L}^{-1}(\dfrac{s^{-1} + s^2}{s^3 + 8s^2 + 8s + 9})$.

d. $y = \mathcal{L}^{-1}(\dfrac{1 + s^2 + 8s}{s^3 + 8s^2 + 8s + 9})$.

e. $y = \mathcal{L}^{-1}(\dfrac{1 - s^2 - 8s + 8s^3}{s^3 + 8s^2 + 8s + 9})$.

PART B

10. Solve the initial value problem

$$y'' + 9y = 9u_\pi(t); \quad y(0) = 0,\ y'(0) = 0.$$

11. a. Find a particular solution of the equation

$$y''' - 5y'' + 4y' = 10e^{-t} + 16t.$$

(Note that $\{1, e^{4t}, e^t\}$ form a fundamental set of solutions for the complementary equation.)

b. Find the general solution for the equation.

12. a. Find a fundamental set of solutions of the equation

$$y''' - y' = 0.$$

b. Using the method of variation of parameters, find a particular solution of the equation

$$y''' - y' = g(t).$$

(Hints: Since you do not know g(t), you must indicate what to do in terms of it.)