Name: ______________________ **Instructor:** ____________

Mathematics 325
Fall Semester 2000
Test 3, Version A
November 16, 2000

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1. This Examination contains twelve problems worth a total of 100 points. The test will be exactly 75 minutes in length.

2. For each multiple choice question, please mark an X on the correct answer on the answer sheet. **Do not circle it.**

3. On problem 10, 11 and 12, show your work, indicating clearly in the space provided how you arrive at your answer; otherwise no credit will be given.

4. Calculators, books and notes are not allowed.

5. Hand in the entire test.

Sign the pledge: "On my honor, I have neither given nor received unauthorized aid on this Exam":

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GOOD LUCK

PART A (5 points each)

1. For the system

$$\frac{d\mathbf{x}}{dt} = \begin{pmatrix} 0 & 5 \\ 3 & 0 \end{pmatrix} \mathbf{x},$$

the critical point $(0, 0)$ is an

a. asymptotically stable node.

b. unstable node.

c. asymptotically stable spiral point.

d. unstable spiral point.

e. unstable saddle point.

2. For the system

$$\frac{d\mathbf{x}}{dt} = \begin{pmatrix} 0 & -5 \\ 3 & 0 \end{pmatrix} \mathbf{x},$$

the critical point $(0, 0)$ is

a. a center.

b. an asymptotically stable spiral point.

c. an unstable spiral point.

d. an asymptotically stable node.

e. an unstable node.

3. For the system

$$\frac{d\mathbf{x}}{dt} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

a. $(-2, -1)$ is a critical point and is an unstable proper node.

b. $(-2, -1)$ is a critical point and is an unstable improper node.

c. $(1, 1)$ is a critical point and is an unstable improper node.

d. $(0, 0)$ is a critical point and is an unstable improper node.

e. $(0, 0)$ is a critical point and is an unstable proper node.

4. For the non-linear system

$$\begin{aligned} \tfrac{dx}{dt} &= -x - y + x^3y - x^2 \\ \tfrac{dy}{dt} &= x - y + xy - x^2y^3, \end{aligned}$$

the critical point $(0, 0)$ is

a. a center.

b. an asymptotically stable node.

c. an unstable node.

d. an asymptotically stable spiral point.

e. an unstable spiral point.

5. Consider the almost linear system

$$\begin{aligned} \tfrac{dx}{dt} &= -x - y + x^2y - y^2 \\ \tfrac{dy}{dt} &= 3x + y + 5xy - xy^2. \end{aligned}$$

The critical point $(0, 0)$ is

a. stable for both the corresponding linear system and the non-linear system.

b. asymtotically stable for both the corresponding linear system and the non-linear system.

c. stable for the corresponding linear system and asmptotically stable for the non-linear system.

d. stable for the corresponding linear system, but the stability for the non-linear system cannot be determined from the corresponding linear system.

e. stable for the corresponding linear system but not stable for the non-linear system.

6. Consider the critical point $(0, 0)$ for the almost linear system

$$\frac{dx}{dt} = x - y + x^2 - y^2$$
$$\frac{dy}{dt} = x + 3y + xy - x^2y.$$

a. The critical point is a spiral point.

b. The critical point is an improper node.

c. The critical point is a node

d. The stability of the critical point cannot be determined from the corresponding linear system.

e. The type of the critical point cannot be determined from the corresponding linear system.

7. Consider the non-linear system

$$\frac{dx}{dt} = 2x - xy$$
$$\frac{dy}{dt} = 3y + xy.$$

The critical point $(-3, 2)$ is a

a. saddle point

b. aymptotically stable node.

c. unstable node.

d. unstable spiral point

e. The stability of the critical point cannot be determined from the corresponding linear system.

8. Except for those on the x-axis and y-axis, the trajectories of the non-linear system

$$\frac{dx}{dt} = x(1+2y)$$
$$\frac{dy}{dt} = y(-2+5x),$$

are described by the equation

a. $-2y + 3xy - x = C$, where C is a constant.

b. $\ln y - 5y = \ln x + 2x + C$, where C is a constant.

c. $y = \frac{C}{x^2}e^{5x}$, where C is a constant.

d. $\ln y + 2y = -2\ln x + 5x + C$, where C is a constant.

e. $y + y^2 = -2x + \frac{5}{2}x^2 + C$, where C is a constant.

9. The system

$$\frac{dx}{dt} = 8x^3 - 4xy^2$$
$$\frac{dy}{dt} = -x^2y - y^3$$

has $(0,0)$ as a critical point. Using $V(x,y) = x^2 - 4y^2$ as a Liapunov function to test the stability of $(0,0)$, what conclusion can we get?

a. $(0,0)$ is an asymptotically stable critical point.

b. $(0,0)$ is a stable critical point.

c. $(0,0)$ is an unstable critical point.

d. $(0,0)$ is a stable but not asymptotically stable critical point.

e. The stability of $(0,0)$ cannot be determined by Liapunov's method.

PART B (15 points each)

10. Find the approximate value of the solution of the initial value problem

$$y' = 2y + 2t, \quad y(0) = 1$$

at $t = 2$ using the Euler method with step size $h = 0.5$.

11. Consider the non-linear system: $\frac{dx}{dt} = y - 1, \qquad \frac{dy}{dt} = 2e^{-x} - 2y.$

a. Find the critical point of this system. (It happens that there is only one critical point for this system.)

b. Write down the corresponding linear system for the critical point. Make the transformation $u = x - x_0$ and $v = y - y_0$ to move the critical point to the origin of the uv-plane, where (x_0, y_0) is a critical point of the original system.

c. Sketch a typical trajectory on the uv-plane for the corresponding **linear** system and indicate the direction of motion for increasing t.

d. Discuss the type and stability of the critical point for the non-linear system.

12. Using Liapunov function of the form $V(x,y) = ax^2 + cy^2$ to test the stability of the critical point $(0,0)$ of the system

$$\frac{dx}{dt} = -4x^3 - xy^2$$
$$\frac{dy}{dt} = 8x^2y - y^3.$$

You must state the correct part of the Liapunov's theorem and verify its conditions.