Name: ______________________________

Mathematics 438
Fall Semester 2000
Midterm Exam 1 (take-home)
Due October 30, 2000

Sign the pledge: "On my honor, I have neither given nor received unauthorized aid on this Exam":

____

GOOD LUCK

1. (20 points) A rat runs around the circle of radius 10 feet centered at $(0,0)$ counter-clockwise with unit speed beginning at $(10,0)$; its position at time s is therefore

$$R(s) = (10\cos\frac{s}{10}, 10\sin\frac{s}{10}).$$

A cat starts at the same time from $(0,0)$ and runs with unit speed with unit speed toward the rat at the times.

(1) Find a formula for the position $c(s)$ of the cat at all times (Hint: Is it true $\frac{dc(s)}{ds} = (\cos\frac{s}{10}, \sin\frac{s}{10})$? Find $\frac{dc(s)}{ds}$ and then integrate).
(2) At what time does the cat bite the rat ?

2. (10 points) (1) Check that the curve

$$\alpha(t) = (\tan t, \sec t)$$

for $-\frac{\pi}{2} =\leq t \leq \frac{\pi}{2}$ is a regular curve.
   (2) Compute $k(t)$;

3. (15 points) Let $\alpha : [0, L] \to \mathbb{R}^2$ be a smooth curve parametrized by arc-length.
   (1) If $k(s) \equiv \frac{1}{r_0}$ for some positive constant r_0 and if $\alpha'(s) = (\cos\theta(s), \sin\theta(s))$, then use the Frenet formula to compute $\theta'(s)$;
   (2) Integrate $\theta'(s)$ and $\alpha'(s)$ to find $\alpha(s)$, where $\theta'(s)$ is given by (1);
   (3) Check that if $k(s) \equiv \frac{1}{r_0}$ for some positive constant r_0 then $\alpha([0, L])$ must be a subset of some circle of radius r_0.

4. (15 points) Suppose that we have a differentiable curve in $\mathbb{R}^3$. Consider its orthogonal projection on the xy-plane. Then prove the second curve has the length less than or equal that of the first. What can you say if their lengths are equal ?

5. (20 points) (1) Compute the first fundamental form of the surface

$$X(u, v) = (e^v \cos u, e^v \sin u, \int \sqrt{1 - e^{2v}} dv)$$

   (2) Use the formula (9) of page 162 to compute the curvature of the surface:

$$X(u, v) = (e^v \cos u, e^v \sin u, \int \sqrt{1 - e^{2v}} dv).$$

6. (20 points) Let $X(u, v) = (\cos u, \sin u, u) + v(-\sin u, \cos u, 1)$.
   (1) Compute the first fundamental form;
   (2) Compute the unit normal vector $N(u, v)$ of the surface
   (3) Compute the Gauss curvature of the surface.