Name: **Professor:**

Mathematics 325
Fall Semester 1999
Test 2
October 12, 2000

1. This Examination contains twelve problems worth a total of 100 points. (There are 9 problems in Part A worth a total of 45 points. There are 3 problems in Part B worth a total of 45 points as well. You begin with 10 points). The test will be exactly 75 minutes in length.

2. For each multiple choice question, please mark an X on the correct answer on the answer sheet. **Do not circle it.**

3. On problem 10, 11 and 12, show your work, indicating clearly in the space provided how you arrive at your answer; otherwise no credit will be given.

4. Calculators, books and notes are not allowed.

5. A table of Laplace transforms is supplied at the end of the booklet.

6. Hand in the entire test.

Sign the pledge: "On my honor, I have neither given nor received unauthorized aid on this Exam":

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1. Find the Laplace transform of $f(t) = \delta(t-\pi)\cos t$.
(a) $-e^{\pi s}$ (b) $-e^{-\pi s}$ (c) -1 (d) $e^{-\pi s}$ (e) $e^{-\pi s}\cos s$.
2. Let $F(s) = \frac{1}{s^3(s^2+4)}$ Find the inverse Laplace transform of F.
(a) $t^2 \sin 2t$ (b) $\frac{1}{2}t^2 \sin 2t$ (c) $\frac{1}{4}t^2 \sin 2t$
(d) $\frac{1}{2}\int_0^t (t-\tau)^2 \sin 2\tau d\tau$ (e) $\frac{1}{4}\int_0^t (t-\tau)^2 \sin 2\tau d\tau$
3. Find the inverse of the matrix $\begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$
(a) $\begin{pmatrix} -1 & 3 \\ 2 & -4 \end{pmatrix}$ (b) $\begin{pmatrix} \frac{3}{2} & -2 \\ -\frac{1}{2} & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 2 & 1 \\ \frac{3}{2} & \frac{1}{2} \end{pmatrix}$ (e) $\begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$
4. Which of the following statements is incorrect?
(a) The system $\begin{aligned} x_1 + x_3 &= 0 \\ x_1 + x_2 &= 0 \\ x_2 + x_3 &= 0 \end{aligned}$ has only the trivial solution.
(b) The vectors $\mathbf{x}^{(1)}(t) = \begin{pmatrix} t^2 \\ t \end{pmatrix}$ and $\mathbf{x}^{(2)}(t) = \begin{pmatrix} t \\ 1 \end{pmatrix}$ are linearly dependent on $0 < t < 1$.
(c) The vectors $\mathbf{x}^{(1)} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\mathbf{x}^{(2)} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ are linearly independent.
(d) The vectors $\mathbf{x}^{(1)} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{x}^{(2)} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, $\mathbf{x}^{(3)} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ are linearly independent.
(e) The two vectors $\mathbf{x}^{(1)}(t) = \begin{pmatrix} \sin t \\ \cos t \end{pmatrix}$ and $\mathbf{x}^{(2)}(t) = \begin{pmatrix} -\cos t \\ \sin t \end{pmatrix}$ are linearly independent on $-\infty < t < \infty$.
5. Using elementary row operations, find the number k such that the system
$\begin{aligned} x_1 - 2x_2 + x_3 &= k \\ x_1 - x_2 + 2x_3 &= 1 \\ 2x_1 - 3x_2 + 3x_3 &= 0 \end{aligned}$ has a solution.
(a) $k = 0$ (b) $k = 2$ (c) $k = 1$ (d) $k = 3$ (e) $k = -1$

6. Suppose that $\mathbf{x}^{(1)} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$. Which $\mathbf{x}^{(2)}$ below will make $\mathbf{x}^{(1)}, \mathbf{x}^{(2)}\}$ linearly independent?
(a) $\mathbf{x}^{(2)} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$ (b) $\mathbf{x}^{(2)} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ (c) $\mathbf{x}^{(2)} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$
(d) $\mathbf{x}^{(2)} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$ (e) $\mathbf{x}^{(2)} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$
7. Compute the eigenvalues of the matrix $\begin{pmatrix} 4 & 100 & 99 \\ 0 & 3 & 2 \\ 0 & 2 & 3 \end{pmatrix}$.
(a) $100, 99, 4$ (b) $4, 5, 1$ (c) $4, 3, 2$ (d) $3+2i, 3-2i, 4$ (e) $3, 2, 3i$

8. Suppose A is a 2×2 real matrix with a complex eigenvalue $r = 3 + 2i$, and $\begin{pmatrix} 1 \\ 2+i \end{pmatrix}$ is the corresponding eigenvector. Which of the following is the general solution to the first order system $\mathbf{x}' = A\mathbf{x}$?

(a) $c_1 e^{3t} \begin{pmatrix} 2\cos 2t - \sin 2t \\ \cos 2t \end{pmatrix} + c_2 e^{3t} \begin{pmatrix} 2\sin 2t + \cos 2t \\ \sin 2t \end{pmatrix}$

(b) $c_1 e^{2t} \begin{pmatrix} \cos 3t \\ 2\cos 3t - \sin 3t \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} \sin 3t \\ 2\sin 3t + \cos 3t \end{pmatrix}$

(c) $c_1 e^{3t} \begin{pmatrix} \cos 2t \\ 2\cos 2t - \sin 2t \end{pmatrix} + c_2 e^{3t} \begin{pmatrix} \sin 2t \\ 2\sin 2t + \cos 2t \end{pmatrix}$

(d) $c_1 e^{-3t} \begin{pmatrix} \cos 2t \\ 2\cos 2t - \sin 2t \end{pmatrix} + c_2 e^{-3t} \begin{pmatrix} \sin 2t \\ 2\sin 2t + \cos 2t \end{pmatrix}$

(e) $c_1 e^{-2t} \begin{pmatrix} \cos 3t \\ 2\cos 3t - \sin 3t \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} \sin 3t \\ 2\sin 3t + \cos 3t \end{pmatrix}$

9. Suppose A is a 3×3 matrix with a repeated eigenvalue $r_1 = r_2 = 5$, and $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ are the corresponding eigenvectors. Then which of the following are solutions of $\mathbf{x}' = A\mathbf{x}$?

(a) $c_1 e^{5t} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_2 e^{5t} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, for all numbers c_1 and c_2

(b) $c_1 e^{5t} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_2 t e^{5t} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, for all numbers c_1 and c_2

(c) $c_1 e^{5t} [t \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}] + c_2 e^{5t} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, for all numbers c_1 and c_2

(d) $c_1 e^{5t} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_2 e^{5t} [t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}]$, for all numbers c_1 and c_2

(e) $c_1 e^{5t} [t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}] + c_2 e^{5t} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, for all numbers c_1 and c_2

10. Find the solution of the initial value problem
$y'' + 4y' + 5y = \delta(t-1),\ y(0) = 0,\ y'(0) = 1.$

11. (a) Find the general solution of $\mathbf{x}' = \begin{pmatrix} 2 & 1 \\ -2 & 5 \end{pmatrix} \mathbf{x}$.

(b) Find the solution that satisfies the initial condition $\mathbf{x}(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

12. Find the general solution of $\mathbf{x}' = \begin{pmatrix} 4 & 0 \\ 1 & 4 \end{pmatrix} \mathbf{x}$.