Math 325, Spring 2000

Review for Midterm

Themes

1. Linear problems

- The set of solutions of a linear homogeneous problem is a vector space.
- Two solutions of an inhomogeneous linear problem differ by a solution of the corresponding homogeneous problem.

2. An existence and uniqueness theorem tells you

- there is a solution to a problem satisfying the hypotheses;
- there is only one solution.

3. Once you have found enough independent solutions to a homogeneous linear problem $Ly = 0$,

- you can find all solutions;
- you can find all solutions to $Ly = g$ starting with a particular solution y_p.

4. Good educated guesses often lead to solutions.

5. Transform a problem to a simple one, solve that, transform that solution back to a solution of the original problem.

Specific Topics

1. Higher order linear ODE

- Existence, uniqueness for initial value problem
- Solutions of nth order homogeneous equation form an n dimensional vector space
- Method of solving constant coefficient homogeneous equations

2. Numerical methods

- Euler's method, estimate for local truncation error
- Improved Euler
- Runge-Kutta

- Stability
  - Importance
  - Tests, methods of judging reliability of computer output (controlling error in dsolve(...,numeric), examining graphical output)

3. Solving ODE with Maple

- symbolic solution using dsolve
- numerical solution using dsolve

4. Systems of first order linear ODE

- Existence, uniqueness
- The solutions of an $n \times n$ linear homogeneous system form an n dimensional vector space
- Constant coefficient systems
  - diagonalizable, real eigenvalues
  - diagonalizable, complex eigenvalues
  - not diagonalizable
    - Jordan Canonical Form
    - only did real eigenvalues in this case
    - know how to find Jordan Canonical Form in 2×2 case
    - know how to use it in general case
- Trajectories
  - interpretation of eigendirections
  - how to tell direction of motion
  - behavior as $t \to \pm\infty$
- Vector field
  - use in determining direction
  - stability, type of critical point at origin
- inhomogeneous system
  - Not necessarily autonomous
  - Trajectory not necessarily independent of t