

Math 325, Spring 2001

Review for Final

Themes

Note: Only themes 6 and 7 are new since the midterm.

1. Linear problems
 - The set of solutions of a linear homogeneous problem is a vector space.
 - Two solutions of an inhomogeneous linear problem differ by a solution of the corresponding homogeneous problem.
2. An existence and uniqueness theorem tells you
 - there is a solution to a problem satisfying the hypotheses;
 - there is only one solution.
3. Once you have found enough independent solutions to a homogeneous linear problem $Ly = 0$,
 - you can find all solutions;
 - you can find all solutions to $Ly = g$ starting with a particular solution y_p .
4. Good educated guesses often lead to solutions.
5. Transform a problem to a simple one, solve that, transform that solution back to a solution of the original problem.
6. Approximate a nonlinear problem by a linear one.
7. Look for solutions of a simple form; try to use them to build all solutions.

Specific Topics

Note: Only topics 5-7 are new since the midterm.

1. Higher order linear ODE
 - Existence, uniqueness for initial value problem
 - Solutions of n th order homogeneous equation form an n dimensional vector space
 - Method of solving constant coefficient homogeneous equations

2. Numerical methods

- Euler's method, estimate for local truncation error
- Improved Euler
- Runge-Kutta
- Stability
 - Importance
 - Tests, methods of judging reliability of computer output (controlling error in `dsolve(...,numeric)`, examining graphical output)

3. Solving ODE with Maple

- symbolic solution using `dsolve`
- numerical solution using `dsolve`

4. Systems of first order linear ODE

- Existence, uniqueness
- The solutions of an $n \times n$ linear homogeneous system form an n dimensional vector space
- Constant coefficient systems
 - diagonalizable, real eigenvalues
 - diagonalizable, complex eigenvalues
 - not diagonalizable
 - * Jordan Canonical Form
 - * only did real eigenvalues in this case
 - * know how to find Jordan Canonical Form in 2×2 case
 - * know how to use it in general case
- Trajectories
 - interpretation of eigendirections
 - how to tell direction of motion
 - behavior as $t \rightarrow \pm\infty$
- Vector field
 - use in determining direction

- stability, type of critical point at origin
 - inhomogeneous system
 - Not necessarily autonomous
 - Trajectory not necessarily independent of t
5. Nonlinear systems of first order ODE
- Autonomous systems
 - critical points, stability
 - using the linearization to determine type, stability, when possible
 - More complicated cases
 - Repeated real eigenvalues (proper node, improper node or spiral point)
 - Imaginary eigenvalues (center or spiral point, indeterminate type)
 - Use of phase portraits, direction fields to determine type, stability
6. PDE, Fourier series
- Separation of variables, especially for the heat equation
 - Fourier series
 - What they are
 - If it looks like one, it is
 - Convergence theorem
 - Sine series, Cosine series
 - More on the heat equation
 - The wave equation
7. The Laplace transform
- Definition, use in solving initial value problems for ODE
 - Discontinuous forcing functions (the Heaviside function or unit step function)
 - Impulse forcing functions (the delta function)