Math 325
Spring, 2001

Reminder: The midterm is Wednesday, March 7. It will cover everything we have done so far through Chapter 7, so Chapter 4, Sections 8.1, 8.3, 8.5, Chapter 7, and Sections 9.1-9.2 in Boyce and DiPrima and Chapters 5, 7, 8, and 12 of *Differential Equations with Maple*. You may bring a summary (one side of an $8\frac{1}{2}'' \times 11''$ sheet of paper, with notes in your writing) to the midterm.

On the course web page, under *Information* you will find a *Review for the Midterm*, which is an outline of the major ideas and topics we have covered. You will also find *Tips for Studying for the Midterm*, which will give you some idea of what to expect on the exam.

The first project will be due Wednesday, April 4. Please be sure to read the guidelines for group projects. The guidelines and the project are posted on the course web site under *Projects*.

ASSIGNMENT 6, due Friday, March 23

Read Boyce and DiPrima, Sections 9.2-9.3.
Do p. 478 #5,24, p. 487 #10 and the *Gallery of Pictures* below.

Gallery of Pictures

This assignment will give you a good feel for the *qualitative* behavior of constant coefficient systems of two ODEs. Use pplane5 to do the plots, and use the Matlab program **multigraf** to put several plots on the same sheet of paper. Be sure to label your plots.

1. Print Figures 1-3 or reproduce them using pplane5. Draw arrowheads on the trajectories to indicate the direction of increasing time. Highlight the eigenspaces. The λ *eigenspace* is a subspace consisting of the origin and all the eigenvectors with eigenvalue λ.

2. Reproduce Figures 4-7 using pplane5. Draw arrowheads on the trajectories to show the direction of increasing time. Highlight eigenspaces.

3. For each system below, use pplane5 to draw a phase portrait for $|x| \leq 2,\ |y| \leq 2$. On the graph, write the eigenvalues of the system matrix, identify any visible eigenspaces and the corresponding eigenvalue, and for (a)-(j), give the portrait its name from the list in Table 9.1.1 on p. 468 of Boyce and DiPrima and its stability type. (*Note*: This list has ten types, since for $r_1 = r_2$, proper and improper nodes are different types. You will use each type once.) Indicate the advance of time by arrowheads on the trajectories. Highlight eigenspaces.

(a) $x' = -x,\ y' = -2y$

(b) $x' = -x,\ y' = -y$

(c) $x' = -x,\ y' = x - y$

(d) $x' = x,\ y' = 2y$

(e) $x' = x,\ y' = y$

(f) $x' = x,\ y' = x + y$

(g) $x' = x,\ y' = -2y$

(h) $x' = -x + 10y,\ y' = -10x - y$

(i) $x' = 10y,\ y' = -10x$

(j) $x' = x + 10y,\ y' = -10x + y$

(k) $x' = 0,\ y' = 0$

(l) $x' = 0,\ y' = x$

(m) $x' = 0,\ y' = -y$

(n) $x' = 0,\ y' = y$

4. For each system in Problem (**??**), plot the graphs of the components and of the solution curve for a "typical" trajectory. (*Note*: You can use the 3d option under graph in pplane5 to plot the solution curve as well as the components.)

5. Systems (k)-(n) in Problem (**??**) are degenerate (at least one eigenvalue is 0), so they are not listed in Table 9.1.1. For each of these, mark equilibrium points and identify the stability type.

6. Show that the origin is a saddle point for the system of the system

$$x' = x,\ y' = 2x - y$$

and draw a phase portrait of the system. Explain the striking similarity with the plot labeled *Singular First Order Equation* (Figure 8).

7. Select a pair of nonorthogonal lines through the origin. Construct a linear system whose matrix has eigenvalues $\lambda_1 = -1,\ \lambda_2 = -2$ with these lines as corresponding eigenspaces. Plot trajectories of your system between the lines. What is the "name" of the system (from Table 9.1.1).

Figure 1: Asymptotically stable proper node

Figure 2: Asymptotically stable improper node

Figure 3: Asymptotically unstable saddle point

Figure 4: Saddle

Figure 5: Node

Figure 6: Center

Figure 7: Spiral

Figure 8: Singular First Order Equation