

## Heading 1 Maple Output Warning 2D Comment 2D Math 2D Output Linear Algebra in Maple

You **must** load the linear algebra package.

```
active1dwith(linalg):
```

Warning, the protected names norm and trace have been redefined and unprotected

```
active1dA := matrix([[1,I],[-I,1]]);
```

```
inert2dA := matrix([[1, I], [-I, 1]]);
```

$$A := \begin{bmatrix} 1 & I \\ -I & 1 \end{bmatrix}$$

Use the command **conjugate** to conjugate the entries of the matrix. You have to use **evalm** (evaluation in matrices) for operations on matrices to be executed.

```
active1devalm(conjugate(A));
```

```
inert2dmatrix([[1, -I], [I, 1]]);
```

$$\begin{bmatrix} 1 & -I \\ I & 1 \end{bmatrix}$$

The command **htranspose** calculates the conjugate transpose (adjoint) of a matrix.

```
active1dhtranspose(A);
```

```
inert2dmatrix([[1, I], [-I, 1]]);
```

$$\begin{bmatrix} 1 & I \\ -I & 1 \end{bmatrix}$$

The comand **map** applies a procedure to each operand of an expression. Use it, for example, to differentiate entries of a matrix.

Here is the matrix of problem 26, p. 357.

```
active1dpsi := matrix([[exp(t),exp(-2*t),exp(3*t)],[-4*exp(t),-exp(-2*t),2*exp(3*t)],  
[-exp(t),-exp(-2*t),exp(3*t)]]);
```

```
inert2dpsi := matrix([[exp(t), exp(-2*t), exp(3*t)], [-4*exp(t), -exp(-2*t),
2*exp(3*t)], [-exp(t), -exp(-2*t), exp(3*t)]]);
```

$$\psi := \begin{bmatrix} e^t & e^{(-2t)} & e^{(3t)} \\ -4e^t & -e^{(-2t)} & 2e^{(3t)} \\ -e^t & -e^{(-2t)} & e^{(3t)} \end{bmatrix}$$

We calculate the derivative of inert2dpsi; $\psi$ .

```
active1dmap(diff,psi,t);
inert2dmatrix([[exp(t), -2*exp(-2*t), 3*exp(3*t)], [-4*exp(t), 2*exp(-2*t),
6*exp(3*t)], [-exp(t), 2*exp(-2*t), 3*exp(3*t)]]);
```

$$\begin{bmatrix} e^t & -2e^{(-2t)} & 3e^{(3t)} \\ -4e^t & 2e^{(-2t)} & 6e^{(3t)} \\ -e^t & 2e^{(-2t)} & 3e^{(3t)} \end{bmatrix}$$

We verify that inert2dpsi; $\psi$  satisfies the differential equation in problem 26.

```
active1dB := matrix([[1,-1,4],[3,2,-1],[2,1,-1]]);
inert2dB := matrix([[1, -1, 4], [3, 2, -1], [2, 1, -1]]);
```

$$B := \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$$

Matrix multiplication is indicated by  $\&^*$ .

```
active1devalm(B*psi);
inert2dmatrix([[exp(t), -2*exp(-2*t), 3*exp(3*t)], [-4*exp(t), 2*exp(-2*t),
6*exp(3*t)], [-exp(t), 2*exp(-2*t), 3*exp(3*t)]]);
```

$$\begin{bmatrix} e^t & -2e^{(-2t)} & 3e^{(3t)} \\ -4e^t & 2e^{(-2t)} & 6e^{(3t)} \\ -e^t & 2e^{(-2t)} & 3e^{(3t)} \end{bmatrix}$$

This is the same as the derivative of inert2dpsi; $\psi$ .

Similarly, we can use map to integrate. For example, to integrate each term of inert2dpsi; $\psi$  from 0 to 1:

```

active1dmap(int,psi,t=0..1);
inert2dmatrix([[exp(1)-1, -1/2*exp(-2)+1/2, 1/3*exp(3)-1/3], [-4*exp(1)+4,
1/2*exp(-2)-1/2, 2/3*exp(3)-2/3], [-exp(1)+1, 1/2*exp(-2)-1/2, 1/3*exp(3)-
1/3]]);

```

$$\begin{bmatrix} e - 1 & -\frac{1}{2} e^{(-2)} + \frac{1}{2} & \frac{1}{3} e^3 - \frac{1}{3} \\ -4e + 4 & \frac{1}{2} e^{(-2)} - \frac{1}{2} & \frac{2}{3} e^3 - \frac{2}{3} \\ -e + 1 & \frac{1}{2} e^{(-2)} - \frac{1}{2} & \frac{1}{3} e^3 - \frac{1}{3} \end{bmatrix}$$

The command **basis** lets you find a basis for the span of a set of vectors. You can use the command **vector** to create the vectors. For example, here are the vectors of problem 10 on p. 366.

```

active1dx[1] := vector([1,2,-2]); x[2] := vector([3,1,0]); x[3]:= vector([2,-
1,1]); x[4] := vector([4,3,-2]);
inert2dx[1] := vector([1, 2, -2]);

```

$$x_1 := [1, 2, -2]$$

```

inert2dx[2] := vector([3, 1, 0]);

```

$$x_2 := [3, 1, 0]$$

```

inert2dx[3] := vector([2, -1, 1]);

```

$$x_3 := [2, -1, 1]$$

```

inert2dx[4] := vector([4, 3, -2]);

```

$$x_4 := [4, 3, -2]$$

```

active1dbasis([x[1],x[2],x[3],x[4]]);
inert2d[x[1], x[2], x[3]];

```

$$[x_1, x_2, x_3]$$

To write  $\text{inert2dx}[4];x_4$  as a linear combination of  $\text{inert2dx}[1],x[2],x[3];;x_1, x_2, x_3$ :

```
active1dv := evalm(x[4]-a*x[1]-b*x[2]-c*x[3]);
inert2dv := vector([4-a-3*b-2*c, 3-2*a-b+c, -2+2*a-c]);
```

$$v := [4 - a - 3b - 2c, 3 - 2a - b + c, -2 + 2a - c]$$

We need to extract the components of  $\text{inert2dv};v$ , then set them equal to 0 and solve the resulting equations. The  $i$ th component is  $v[i]$ .

```
active1dsolve({v[1],v[2],v[3]},{a,b,c});
inert2d{a = 1, c = 0, b = 1};
```

$$\{a = 1, c = 0, b = 1\}$$

The command **eigenvects** allows you to calculate the eigenvectors of a matrix. The answer comes in the form of a list. Each item in the list consists of an eigenvector, its multiplicity, and a basis for the eigenspace. As an example, here is problem 17 on p. 367.

```
active1dC := matrix([[-2,1],[1,-2]]);
inert2dC := matrix([[-2, 1], [1, -2]]);
```

$$C := \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$$

```
active1de := eigenvects(C);
inert2de := [-3, 1, {vector([-1, 1])}], [-1, 1, {vector([1, 1])}];
```

$$e := [-3, 1, \{-1, 1\}], [-1, 1, \{1, 1\}]$$

To make a matrix  $\text{inert2dT};T$  whose columns are the eigenvectors we just found, we first have to extract the eigenvectors. For example,  $[-1,1]$  is the first thing in  $\{-1,1\}$ , which is the third in the second element of  $\text{inert2de};e$ , i.e., it is  $e[2][3][1]$ .

```
active1dT := transpose(matrix([e[1][3][1],e[2][3][1]]));
inert2dT := matrix([[-1, 1], [1, 1]]);
```

$$T := \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

The command **diag** produces a diagonal matrix with the given entries on the diagonal.

```
active1dDiag := diag(-1,-3);  
inert2dDiag := matrix([[ -1, 0], [0, -3]]);
```

$$Diag := \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}$$

The command **inverse** finds the inverse of a matrix. We use it in checking that  $\text{inert2dT}^{(-1)} * C * T = Diag$ ;  $T^{(-1)} C T = Diag$ .

```
active1devalm(inverse(T)*C*T);  
inert2dmatrix([[ -3, 0], [0, -1]]);
```

$$\begin{bmatrix} -3 & 0 \\ 0 & -1 \end{bmatrix}$$

**Warning:** You cannot use either Psi or D as a name in Maple, because both are built in.