Maple Output Maple Plot Warning 2D Math 2D Output
Math 325: Differential Equations Name:________________________
Midterm *March 7, 2001*

You may use your own calculator and an $8\frac{1}{2}'' \times 11''$ sheet of paper with notes in your writing on one side. You may not use anything else. You may not pass a calculator or notes to another person.

Show all your work. Erase or cross out any work you do not want graded. There are four questions on 8 pages plus the bonus question page.

1. (25 points) a) Solve

$$y''' + 3y'' + 2y' = 0, \qquad y(0) = -1,\ y'(0) = -2,\ y''(0) = -4.$$

b) Write a sequence of Maple commands designed to solve the initial value problem in a) and to determine the floating point value of $y(13)$.

2. (20 points) Solve

$$\mathbf{x}' = \begin{bmatrix} -193 & 1 \\ 0 & -193 \end{bmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{bmatrix} \sqrt{491} \\ 19^5 \end{bmatrix}.$$

3. (25 points) Consider this snapshot of Doug's Maple session. Then answer the questions on the next page.

active1dwith(linalg):
Warning, new definition for norm
Warning, new definition for trace
active1divp := {diff(x(t),t)=5*x(t)/4+3*y(t)/4, diff(y(t),t)= 3*x(t)/4+5*y(t)/4,x(0)=a,y(0)=b}:

active1dsolivp := dsolve(ivp,{x(t),y(t)});
inert2dsolivp := {x(t) = (1/2*b+1/2*a)*exp(2*t)+(-1/2*b+1/2*a)*exp(1/2*t), y(t) = (1/2*b+1/2*a)*exp(2*t)-(-1/2*b+1/2*a)*exp(1/2*t)}; solivp := {x(t)=$(\frac{1}{2}\,b+\frac{1}{2}\,a)\,e^{(2\,t)}+$
$(-\frac{1}{2}\,b+\frac{1}{2}\,a)\,e^{(1/2\,t)}$,
y(t) = $(\frac{1}{2}\,b+\frac{1}{2}\,a)\,e^{(2\,t)}-(-\frac{1}{2}\,b+\frac{1}{2}\,a)\,e^{(1/2\,t)}$}
active1dparasol := unapply(subs(solivp,[x(t),y(t),t=trange]),a,b,trange):
active1d plot({seq(seq(parasol(a,b,-1..1),a=-2..2),b=-2..2)});

midterm301.eps

a) What system of ODE is he studying?

b) What are the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} \frac{5}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{5}{4} \end{bmatrix}?$$

c) On the plot on the previous page, identify eigenvectors (if relevant) and the direction of increasing time on the trajectories.

d) What is the type and stability of the origin as a critical point of the system under study?

4. Ellen decided to use Maple to study the initial value problem

$$y' = x^2 + y^2, \qquad y(0) = 1.$$

Here is a snapshot of her Maple session.
active1dwith(plots):
Warning, the name changecoords has been redefined
active1deqn := diff(y(x),x)=y(x)$^2 + x^2$;
inert2deqn := diff(y(x),x) = y(x)$^2 + x^2$; $eqn := \frac{\partial}{\partial x}\,\mathrm{y}(x) = \mathrm{y}(x)^2 + x^2$
active1dphi[0] := dsolve({diff(y(x),x)=y(x)$^2, y(0) = 1\}, y(x))$;
inert2dphi[0] := y(x) = -1/(x-1);

$$\phi_0 := \mathrm{y}(x) = -\frac{1}{x-1}$$

active1dphi[1] := dsolve({diff(y(x),x)=1+y(x)$^2, y(0) = 1\}, y(x))$;
inert2dphi[1] := y(x) = tan(x+1/4*Pi);

$$\phi_1 := \mathrm{y}(x) = \tan(x + \frac{1}{4}\,\pi)$$

active1devalf(Pi/4);
inert2d.7853981635;

$$.7853981635$$

active1dsoln := dsolve({eqn,y(0)=1},y(x),numeric);
inert2dsoln := proc (rkf45$_x$)$locali, rkf45_s, outpoint, r1, r2; globalloc_control, loc_y0, loc_y1; option`Copyrig$
$evalf(rkf45_x); ifabs(-outpoint) < abs(loc_control[2]-outpoint)ornotmember(loc_control[6], \{-2, -1, 1$
$copy(array(1..26, [(1) = 1, (2) = 0., (3) = 0., (4) = .1e-7, (5) = .1e-7, (6) =$
$1, (7) = .1e-8, (8) = 30000, (9) = 1000, (10) = 0, (11) = 0, (12) = 0, (13) =$
$0, (14) = 0, (15) = 0, (16) = 0, (17) = 0, (18) = 0, (19) = 0, (20) = 1., (21) =$
$0, (22) = 0, (23) = 0, (24) = 0, (25) = 0, (26) = 0])); loc_y0 := copy(array(1..1, [(1) =$
$1.])); loc_y1 := copy(array(1..1, []))endif; ifabs(-outpoint) <> 0thenloc_control[3] :=$
$outpoint; ifDigits <= evalhf(Digits)thenrkf45_s := traperror(evalhf(`dsolve/numeric_solnall_rkf45$
$lasterrorthenr1 := searchtext('evalhf', convert(op(1, [rkf45_s]), name)); r2 := searchtext('hardware'$
$0orr2 <> 0then`dsolve/numeric_solnall_rkf45`(loc_F, loc_control, loc_y0, loc_y1, loc_F1, loc_F2, loc_F3, loc_F4, l$
$rkf45_x, seq(ord[i+1] = loc_y0[i], i = 1..1)]endproc; soln :=$ **proc**(*rkf45_x*) ... **end proc**

active1dsol := u -¿ subs(soln(u),y(x)):
active1dplotsol := plot(sol,0..0.965,color=black,numpoints=1000):
active1dplotphi[0] := plot(rhs(phi[0]),x=0..0.995,linestyle=4,color=black,numpoints=2000):

active1dplotphi[1] := plot(rhs(phi[1]),x=0..0.78,linestyle=7,color=black,numpoints=1000):

active1ddisplay({plotsol,plotphi[0],plotphi[1]},title=`Plot 1`);

midterm401.eps

active1dcurve := a -¿ odeplot(dsolve({eqn,y(0)=a },y(x),numeric),[x,y(x)],0..0.885,numpoints=1000,co

inert2dcurve := proc (a) options operator, arrow; odeplot(dsolve({eqn, y(0) = a},y(x),numeric),[x, y(x)],0 .. .885,numpoints = 1000,color = black) end proc; curve := a→ odeplot(dsolve({*eqn*, y(0) = *a*}, y(x), *numeric*), [x, y(x)], 0...885,
numpoints = 1000, *color* = *black*)
active1ddisplay({seq(curve(1+0.1*j),j=-1..1)},title=‘Plot 2‘);

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a) Why does she introduce the functions ϕ_0 and ϕ_1?

b) Label the three graphs in Plot 1.

c) How does the solution of of the initial value problem being studied behave as x increases? Justify your answer.

d) Suppose the initial value is changed from 1 to a value near 1. How will the solution change? Justify your answer.

e) Label the three graphs in Plot 2. How does this plot relate to d)?

Name:________________________

This page is due by the start of class on Friday, March 9.

Bonus Problem(10 points) In Problem 1b), you wrote a sequence of Maple commands to solve the initial value problem

$$y''' + 3y'' + 2y' = 0, \qquad y(0) = -1,\ y'(0) = -2,\ y''(0) = -4$$

and determine the floating point value of $y(17)$. Write your sequence of Maple commands here, take this sheet with you, and try the commands to see if they work. If they do not, figure out what is wrong and correct it. Turn in this sheet (with an explanation of what is wrong) together with a Maple worksheet showing a correct solution.

Bonus Question(3 points for a thoughtful answer) Have you found the worksheets we have done in class valuable? Why or why not? If you have found them valuable, do you think additional class time should be spent on worksheets? Why or why not?