1. This examination contains twelve problems worth a total of 100 points. The test will be exactly 60 minutes in length.

(Part A: 6 x 10 points = 60 points, Part B: 2 x 15 points = 30 points and you begin with 10 points)
2. For each multiple choice question, please mark an X on the correct answer on the answer sheet.

Do not circle it. 3. On problem 11 and 12, show your work, indicating clearly in the space provided how you arrive at your answer; otherwise no credit will be given. 4. Calculators, books and notes are not allowed. 5. A table of Laplace transforms is supplied at the end of the booklet. 6. Hand in the entire test.

Sign the pledge: "On my honor, I have neither given nor received unauthorized aid on this Exam":
____________________________ **GOOD LUCK**

PART A 1. Find the general solution of

$$y''' - 4y'' + 5y' - 2y = 0.$$

a. $C_1e^t + C_2e^{2t}$. b. $C_1e^t + C_2te^t + C_3e^{2t}$. c. $C_1e^t + C_2e^{-t} + C_3e^{2t}$. d. $C_1e^t + C_2te^t + C_3e^{-2t}$. e. $C_1 \cos t + C_2 \sin t + C_3e^{-2t}$.

2. Consider the initial value problem

$$ty'' + 2y' + (\cos t)y = t^2.$$

Where are the intervals in which solutions are sure to exist? a. $-\infty < t < \infty$. b. $t > 0$ only. c. $t < 0$ only. d. $t > 0$ or $t < 0$. e. $t = 0$.

3. Consider the nonhomogeneous equation

$$y^{iv} + 2y'' + y = 3 \cos t - 14t^2$$

Find the suitable form of a particular solution if the method of undetermined coefficients is to be used.

a. $t^2(A \cos t + B \sin t) + Ct^2 + Dt + E$. b. $t(A \cos t + B \sin t) + Ct^2 + Dt + E$. c. $A \cos t + B \sin t + Ct^2$. d. $A \cos t + Bt^2 + Ct + D$. e. $t^2(A \cos 2t + B \sin 2t) + t(Ct^2 + Dt + E)$.

4. Find the general solution of the equation

$$y'' - y = 6e^{2t}.$$

a. $y(t) = C_1e^t + C_2e^{-t} + e^{2t}$. b. $y(t) = C_1e^t + C_2e^{-t} + 2e^{2t}$.

c. $y(t) = C_1e^t + C_2e^{-t} + te^{2t}$. d. $y(t) = C_1e^t + C_2e^{-t} - e^{2t}$. e. $y(t) = 2e^{2t}$.

5. Which set of functions is linearly dependent? a. $f_1(t) = 1, f_2(t) = t$. b. $f_1(t) = t + 1, f_2(t) = t$.

c. $f_1(t) = t + 1, f_2(t) = t, f_3(t) = t^2$.

d. $f_1(t) = t + 1, f_2(t) = t, f_3(t) = 2t + 3$. e. $f_1(t) = t + 1, f_2(t) = t, f_3(t) = t^2 + t$.

6. Find the Laplace transform of

$$f(t) = \delta(t - \pi)t^2.$$

a. $\frac{2}{s^3}e^{-\pi s}$.

b. $-\pi^2e^{\pi s}$. c. π^2. d. $\pi^2e^{\pi s}$.

e. $\pi^2e^{-\pi s}$.

7. Find the Laplace transform of the function

$$f(t) = \{ \quad 0, \qquad 0 < t < 1t^2 - 2t, \quad t \geq 1.$$

a. $e^{-s}(\frac{2}{s^3} - \frac{1}{s})$. b. $e^{-s}(\frac{2}{s^3} + 2\frac{1}{s})$. c. $e^{-s}(\frac{2}{s^3} - 1)$. d. $e^{-s}(\frac{1}{s^2} - \frac{1}{s})$. e. $\frac{2e^{-s}}{s^3} + 1$.

8. Find the inverse Laplace transform of $f(s) = \frac{s}{s^2-2s+5}$.

a. $e^t \cos 2t$ b. $e^t \cos 2t + e^t \sin 2t$ c. $e^t \cos 2t - e^t \sin 2t$ d. $e^{-t} \cos 2t + e^{-t} \sin 2t$ e. $e^t \cos 2t + \frac{1}{2}e^t \sin 2t$

9. Find the solution to the following initial value problem

$$y^{iv} + y'' + y = 1; \quad y(0) = 0,\ y'(0) = 0, y''(0) = 0, y'''(0) = 0$$

a. $y = 1$. b. $y = 0$. c. $y = L^{-1}(\frac{s}{s^4+s^2+1})$. d. $y = L^{-1}(\frac{1}{s^4+s^2+1})$.

e. $y = L^{-1}(\frac{1}{s(s^4+s^2+1)})$.

10. Find the solution to the following initial value problem

$$y'' + y = \delta(t - \pi); \quad y(0) = 0,\ y'(0) = 0.$$

a. $y = u_\pi(t)\sin(t-\pi)$. b. $y = e^{-\pi t}\sin t$. c. $y = u_\pi(t)\cos(t-\pi)$. d. $y = \sin t$.

e. $y = 0$.

PART B

11. Use the method of variation of parameters to find the general solution of the equation

$$y''' - y' = 2e^t.$$

12. Solve the initial value problem

$$y'' + 4y = 4u_\pi(t); \quad y(0) = 0, \ y'(0) = 1.$$