ath 335, Test 1, Fall (1992)

The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. The problems are worth 20 points apiece; in the case of a problem with multiple sections, the credit will be divided equally between the parts. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (texbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

1

1. 1

"(1)"

1 (a) State the principle of mathematical induction. (b) *Using this principle*, give a careful proof of the formula

$$1 + 2 + \cdots + n = \frac{1}{2}n(n+1)$$

2 (a) State the Completeness Axiom for the field $\mathbb{R}$ of real numbers. (b) Give an example of a nonempty set S of rational numbers such that $sup(S)$ exists and is a real number which is *not* rational.

3 Let s and t be real numbers, and let (s_n) and (t_n) be sequences of real numbers. a Give the definition of what it means for $\lim s_n$ to be equal to s. b Suppose that $\lim s_n = s$ and $\lim t_n = t$. Prove carefully from the definition that $\lim(s_n + t_n) = s + t$.

4 For each of the four sequences $(s_n)_{n=1}^{\infty}$ below determine whether it converges and, if it converges, give its limit. It is not necessary to give formal proofs that your calculations are correct. a $s_n = (7n^2 - 3)/(2n^2 + 5)$ b $s_n = (1 + 1/n)^4$ c $s_n = \sin(n)/n$ d $s_n = (2^{n+1} + 3)/(2^n + 1)$

5 Consider the sequence

$$(s_n) = \left((-1)^n + \frac{1}{n}\right)_{n \geq 1}$$

(it might be helpful for you to write out the first few terms to get a sense of how the sequence behaves). a Compute $sup\{s_n : n \geq 1\}$. b Compute $inf\{s_n : n \geq 1\}$. c Compute lim $sups_n$. d Compute lim $infs_n$. e Does the sequence converge? Justify your answer.