ath 335, Test 2, Fall (1992)

The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. The problems are worth 20 points apiece; in the case of a problem with multiple sections, the credit will be divided equally between the parts. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (texbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

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1. 1

"(1)"

1 (a) Define the notion of “Cauchy sequence”. (b) Prove that every convergent sequence is a Cauchy sequence.

2 State and prove the Bolzano-Weierstrass theorem for sequences of real numbers.

3 Which of the following series converge? a $\sum \frac{1}{n^4+3n^2}$ b $\sum \left(\frac{3}{\pi}\right)^n$ c $\sum \left(\frac{\pi}{3}\right)^n$ d $\sum \frac{(-1)^n \cos(n+1)}{n^{3/2}}$

4 For each of the statements below, either give a proof (perhaps by referring to a theorem) or give a counterexample. a The set $S = [0,1] \cup (2,3]$ is the set of subsequential limits of some sequence (s_n) of real numbers. b A series $\sum a_n$ converges if and only if $\lim a_n = 0$.

5 Find the set S of subsequential limits of the sequences (s_n) below: a (s_n) is an enumeration of **all** rational numbers in $\mathbb{R}$. b

$$s_n = \{\, n \,, if$$

n¡10^{10}

$1+\frac{(-1)^n}{n}, if$ n$\geq 10^{10}$ and n is even

$\sin(\frac{\sqrt{2}}{n}), if$ n$\geq 10^{10}$ and n is odd.