ath 335, Test 3, Fall (1992)

The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. The problems are worth 20 points apiece; in the case of a problem with multiple sections, the credit will be divided equally between the parts unless otherwise indicated. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (texbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

Note: all functions below are *real-valued*.

1

1. 1

”(1)” dom

1 Consider the series $\sum_{i=1}^{\infty} \frac{(-1)^n}{n^{1/10}}$. a Does this series converge? b Does this series converge absolutely? In each case justify your answer.

2 (a) [5 pts] Give the definition of what it means for a function f to be *continuous*. b [10 pts] Let f be a continuous function with domain (a, b). Suppose that $f(r) = 0$ for each rational number $r \in (a, b)$. Show that $f(x) = 0$ for all $x \in (a, b)$. c [5 pts] Let f be the function with domain (a, b) defined by letting $f(x) = 0$ for rational numbers x and $f(x) = 1$ for irrational numbers. Show that f is not continuous.

3 Suppose that f is a continuous function on the closed interval $[a, b]$. Prove that f is bounded.

4 (a) State the Intermediate Value Theorem. (b) Suppose that f is a continuous function on $\mathbb{R}$ and that $f(a)f(b) < 0$ for some a, $b \in \mathbb{R}$. Show that there exists a number x between a and b such that $f(x) = 0$.

5 Determine whether or not the following functions are uniformly continuous on the specified sets.

a $f(x) = \frac{x-1}{x+2}$ on $[0, 1]$. b $f(x) = \sin(\frac{1}{x^2+1})$ on $(0, 1)$. c $f(x) = \max(-1, -1/x)$ on $(0, 1)$. c $f(x) = \sum_{i=0}^{\infty} x^i$ on $(-1, 0]$. (Note that the series converges for x in this interval.)