ath 335, Final Exam, Fall (1993)

The test will be 2 hours in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. Except for problem 5, each problem is worth 20 points. Unless otherwise indicated, credit on a multipart problem is divided evenly between the parts. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (textbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

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1 (a) State the principle of mathematical induction. (b) Show that for every integer $n \geq 1$ there is an equality

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2} \ .$$

2 (a) Explain what is meant by the "denseness of $\mathbb{Q}$". (b) Suppose that f is a continuous function on $\mathbb{R}$ such that $f(x) = 0$ for all $x \in \mathbb{Q}$. Show that f is the zero function.

3 Prove that every bounded monotone sequence converges.

4 (a) State the comparison test for the convergence of series. (b) Show that if $\sum a_n$ is a convergent series of nonnegative numbers and $p > 1$, then $\sum a_n^p$ converges.

5 (a) (20 pts) Prove that every sequence (s_n) has a monotone subsequence. (b) (10 pts) State and prove the Bolzano-Weierstrass theorem.

6 Suppose that f is uniformly continuous on a set S and that (s_n) is a Cauchy sequence in S. Prove that $(f(s_n))$ is a Cauchy sequence.

7 a Suppose that (s_n) is a sequence and s is a real number. Give the definition of what it means for the limit of (s_n) to be equal to s. b Suppose that f is a function defined in some interval containing the real number a. Define in terms of sequences what it means for f to be continuous at a. c Suppose that f is a function defined in some interval containing the real number a. Define in terms of epsilons and deltas what it means for f to be continuous at a. c Let f be a function defined for all real numbers, and let b be a real number. Define what "$\lim_{x\to\infty} f(x) = b$" means.