ath 335, Test 1, Fall (1993)

The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. The problems are worth 20 points apiece; in the case of a problem with multiple sections, the credit will be divided equally between the parts unless otherwise specified. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (texbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

1

1. 1

"(1)"

1 (a) State the principle of mathematical induction. (b) *Using this principle*, give a careful proof of the formula

$$1^2 + 2^2 + \cdots + n^2 = n(n+1)(2n+1)/6$$

2 (a) State the Completeness Axiom for the field $\mathbb{R}$ of real numbers. (b) Explain what is meant by the "Denseness of $\mathbb{Q}$".

3 The *Archimedean Property* states that if $a > 0$ and $b > 0$ are real numbers, then for some positive integer n we have $na > b$. Use the Completeness Axiom to prove this property.

4 Prove using the definition of limit that $\lim_{n\to\infty} a^n = 0$ if $|a| < 1$.

5 Consider the set $\{s \in \mathbb{Q} | s^2 < 3\}$. a (5 pts) Does this set have an upper bound in $\mathbb{Q}$? If so, give one. b (5 pts) Does this set have a least upper bound in $\mathbb{Q}$? If so, determine what it is. c (10 pts) Explain how your answers are consistent with the Completeness Axiom.