ath 335, Test 2, Fall (1993)

The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. The problems are worth 20 points apiece; in the case of a problem with multiple sections, the credit will be divided equally between the parts unless otherwise indicated. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (texbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

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1 It is known that all bounded monotone sequences converge. Prove this for the special case of bounded *nondecreasing* sequences.

2 Consider the sequence (s_n) with terms given by the formula

$$s_n = \left((-1)^n + \frac{1}{n}\right)\left(\frac{n^2+1}{n^2+4}\right) \quad .$$

a Give an example of a monotone subsequence of (s_n). b Determine the set of subsequential limits of (s_n). c Determine $\limsup s_n$ and $\liminf s_n$. d Is this sequence bounded? e Does this sequence converge?

3 State and prove the "Root Test". Be sure to include criteria both for convergence and divergence.

4 Show that if $\sum a_n$ and $\sum b_n$ are convergent series of nonnegative numbers, then $\sum \sqrt{a_n b_n}$ converges. (*Hint*: Show that $\sqrt{a_n + b_n} < a_n + b_n$ for all n.)

5 Determine whether or not the following series converge. Justify your answers. a $\sum_{n=1}^{\infty} 3^n/n!$ b $\sum_{n=1}^{\infty} (-1)^n/n^2$ c $\sum_{n=1}^{\infty} 1/n^{2n}$ d $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} + \frac{1}{2^n}\right)$