ath 335, Test 3, Fall (1993)

The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. The problems are worth 20 points apiece; in the case of a problem with multiple sections, the credit will be divided equally between the parts unless otherwise indicated. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (texbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

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1 Let f be a continuous real-valued function on the closed interval $[a, b]$. Prove that f is a bounded function.

2 Determine whether or not the following functions are *uniformly continuous* on the indicated sets. Justify your answers. a $x \in (-\pi, \pi), \quad f(x) = \{ \cos(1/x)x \neq 0$

$0x = 0$ b $x \in (1, 2), \quad f(x) = (\sqrt{x} - 1)/(x - 1)$ c $x \in [-1, 1], \quad f(x) = |x|$ d $x \in (0, 1), \quad f(x) = 1/x$

3 Prove that if f is continuous at x_0 and g is continuous at $f(x_0)$ then the composite function $g \circ f$ is continuous at x_0.

4 Give an example of a series $\sum a_n$ such that $\sum a_n$ converges but $\sum |a_n|$ does not converge. Justify your answer.

5 (a) State the Intermediate Value Theorem. (b) Let f be the function given by the formula $f(x) = x^5 + 9x^3 + 6x$. Show that for any real number y there is at least one real number x such that $f(x) = y$. (Hint: Observe that $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to-\infty} f(x) = -\infty$.)