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This exam will be 2 hours in length. It consists of 15 multiple-choice problems, worth
For each of these problems, mark an **X** over the letter below that corresponds to the
The instructor will distribute blue books to use for scratch work.

At the end of the exam please hand in a **complete** test booklet (answer sheet + questi
Math 335–Final Exam December 1994 your name on it. Please also leave the blue book with your scratch work in it with th
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Which **ONE** of the following statements about sequences is **CORRECT**? Every bounded sequence has a Cauchy subsequence. Every bounded sequence is convergent. Every sequence has a bounded subsequence. Every sequence which is bounded above has a convergent subsequence. Every sequence which has a convergent subsequence is bounded above. Consider the following limit calculations:

(1) $\lim \sqrt{4n^2+n} - 2n = 14$ (2) $\lim\left((-1)^n + 1n\right) = 1$ (3) $\lim\left(2^n n^2 + (-1)^n\right) = \infty$ Which of these calculations are **WRONG**? (2) only (1) and (2) (2) and (3) (1) and (3) (3) only

Which **ONE** of the following sets **CAN** be the set of subsequential limits of a sequence? The union of $[-1,0]$ with $\{1n : n \geq 1\}$ The set of rational numbers in $[0,1]$ The set of irrational numbers in $[0,1]$ $\{1n : n \geq 1\}$ $(-5,5)$ Let (s_n) be a sequence. Which of the following statements expresses the fact that (s_n) does **NOT** converge to the number a? There exists $\epsilon > 0$ such that for any number N there is a positive integer $n > N$ with $|s_n - a| > \epsilon$. For any $\epsilon > 0$ there exists a number N such that for all positive integers $n > N$, $|s_n - a| \geq \epsilon$. There exists a number $\epsilon > 0$, a number N, and a positive integer $n > N$, such that $|s_n - a| > \epsilon$. For any N there exists an $\epsilon > 0$ such that for all positive integers $n > N$, $|s_n - a| > \epsilon$. There exists a number $\epsilon > 0$ and a number N such that, for all positive integers $n > N$, $|s_n - a| > \epsilon$. Suppose that S is a nonempty set of real numbers which is bounded above. Let $s_0 = \sup(S)$. Which **ONE** of the following statements is **FALSE**? For any $\epsilon > 0$ there exists $s \in S$ such that $s_0 + \epsilon \geq s > s_0$. $s_0 = -\inf\{-s : s \in S\}$ For any $\epsilon > 0$ there exists $s \in S$ such that $s_0 - \epsilon < s \leq s_0$. If S is a closed set, then $s_0 \in S$. If $\max(S)$ exists, then $s_0 = \max(S)$.

Suppose that (a_n) and (b_n) are convergent sequences with $\lim a_n = a$ and $\lim b_n = b$. Let (c_n) be the sequence formed by setting $c_n = a_n + (-1)^n b_n$. What, in general (that is, with no special assumptions about a and b), is the set of subsequential limits of (c_n)? $\{a+b, a-b\}$ $\{a, b\}$ $\{a, -b\}$ $\{a, b, a-b, a+b\}$ $\{-a, -b, a, b, \}$ Let $f : R \to R$ be a continuous function, and let x be a real number. Consider the sequence (s_n) defined inductively by setting $s_0 = x$ and letting $s_n = f(s_{n-1})$ for $n > 0$. In other words, (s_n) is the sequence

$x,\ f(x),\ f(f(x)),\ f(f(f(x))),\ \ldots$ Suppose that (s_n) converges and that $\lim s_n = y$. Which **ONE** of the following statements about y is necessarily **TRUE**? $y = f(y)$ $y = x$ $y = f(x)$ The number y cannot exist. Such a sequence never converges. $f(y - x) = 0$ Which **ONE** of the following statements is **TRUE**? If $f : [a,b] \to R$ and $g : R \to R$ are continuous, then the composite function $g \cdot f$ is uniformly continuous. If $f : (a,b) \to R$ is continuous and (s_n) is a Cauchy sequence of points from (a,b), then the sequence $(f(s_n))$ is Cauchy. A uniformly continuous function $f : R \to R$ must be bounded. There exists a uniformly continuous function $f : (0,1) \to R$ which cannot be extended to a continuous function $\tilde{f} : [0,1] \to R$. If $f : (0,1) \to R$ is a continuous function which cannot be extended to a continuous function $\tilde{f} : [0,1] \to R$, then f is not bounded. Which **ONE** of the following statements is **TRUE**? If $f : R \to R$ is a continuous function such that $f(x) = 0$ for every irrational number x, then f is the zero function. Let $f : R \to R$ be the function given by letting $f(x) = \sin(1/x)$ for $x \neq 0$ and setting $f(0) = 1$. Then f is continuous. There exists a continuous function $g : R \to R$ such that $g(12^n) = 0$ for all $n \geq 1$, but $10^{-6} \leq g(0) \leq 10^{-3}$. A function

$f : [0,1] \to R$ is continuous at $x_0 = 12$ if there exists $\epsilon > 0$ such that $|f(x) - f(12)| < \epsilon$ for every $x \in [1,2]$. Suppose that S is a set of real numbers. Then a function $f : S \to R$ is continuous if and only if the function $|f| : S \to R$ is continuous. Which **ONE** of the following statements is **TRUE**? There exists $x \in R$ such that $x = 100\cos x$ There exists a continuous function $f : [0,1] \to R$ whose image is the interval $[0,\infty)$. There exists a continuous function $f : (0,1) \to R$ whose image is the union of the intervals $(1,2)$ and $(3,4)$. The function $f : R \to R$ given by $f(x) = 2 - \cos^2 x$ has a zero. Let $f : (-1,1) \to R$ be a function which is continuous but not uniformly continuous. Suppose that (x_n) is a sequence of points from $(0,1)$ which converges to 1. Then the sequence $(f(x_n))$ does not converge. Consider the power series $\sum x^n \sqrt{n}$. What is its radius of convergence? 1 0 ∞ 2 $\sqrt{2}$ Calculate the sum of the geometric series $\sum_{n=2}^{\infty} 12^n$. (Note that the summation starts with $n = 2$.) 12 1 2 32 52 Consider the two series

$\sum_{n=1}^{\infty} (-1)^n n^{10} (1.001)^n$ and $\sum_{n=1}^{\infty} (-1)^n n^{1/10}$. Which **ONE** of the following statements is **TRUE**. The first series converges absolutely and the second one converges (but not absolutely). Both series converge absolutely. Both series diverge. The second series converges absolutely, but the first one diverges. The second series converges absolutely, and the first one converges (but not absolutely). The full **interval of convergence** of the power series $\sum_{n=1}^{\infty} x^n n$ is $[-1,1)$ $[-1,1]$ $(-1,1)$ $(-1,1]$. $(-\infty,\infty)$. For each positive integer n, define the number d_n to be equal to 1 if the decimal expansion of n ends with a 3, and to be equal to 0 otherwise. Consider the series

$\sum_{n=1}^{\infty} d_n 7^n x^n$ What is its radius of convergence?

7 13 37 73 21