ath 335, Test 1, Fall (1994)

The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. The problems are worth 20 points apiece; in the case of a problem with multiple sections, the credit will be divided equally between the parts unless otherwise indicated. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (texbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

1

1. 1

1

"(1)" dom

1 Prove that if (s_n) converges to s and (t_n) converges to t then $(s_n t_n)$ converges to st; in other words, prove that

$$\lim(s_n t_n) = \lim(s_n) \lim(t_n) \ .$$

2 Prove that if $a > 0$ and $b > 0$ are real numbers, then for some positive integer n, we have $na > b$. (This is the *Archimedean Property*.)

3 (a) State the principle of mathematical induction. (b) Use this principle to show that

$$1^2 + 2^2 + \cdots + n^2 = n(n+1)(2n+1)/6$$

for all natural numbers n.

4 a (10 pts) Define what it means for a sequence (s_n) of real numbers to converge to a real number s. b (5 pts) Give an example of a sequence (x_n) of irrational numbers having a limit $\lim x_n$ that is a rational number. c (5 pts) Give an example of a sequence (r_n) of rational numbers having a limit $\lim r_n$ that is an irrational number.

5 Give the following when they exist. Otherwise assert "NOT EXIST". a $\lim\,(\sqrt{3}-\sqrt{2})^n$ b $\lim\,(5n^2+3)/(2n^2+17)$ c $\lim\,(-1)^n/n$ d $\lim s_n$, where $s_1 = 1$ and $s_{n+1} = (s_n^2+3)/2s_n$. You should take for granted that this sequence does in fact converge.