ath 335, Test 2, Fall (1994)

The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. The problems are worth 20 points apiece; in the case of a problem with multiple sections, the credit will be divided equally between the parts unless otherwise indicated. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (texbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

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"(1)" dom 1 Prove that any bounded nodecreasing sequence (s_n) converges.

2 Prove that any sequence (s_n) has a monotone subsequence. Use this to prove the *Bolzano-Weierstrass theorem* (which states that any bounded sequence has a convergent subsequence).

3 Let (q_n) be an enumeration of the rational numbers in the open interval $(0, 1)$, that is, a sequence which contains every rational number in the interval exactly once. a (5 pts) Describe the set of subsequential limits of (q_n). b (5 pts) Compute $\limsup q_n$ and $\liminf q_n$. c (10 pts) Does there exist a sequence (s_n) which has $(0, 1)$ as its set of subsequential limits? Either give an example of such a sequence or explain clearly why one cannot exist.

4 Determine which of the following series converge.

(a) $\sum n^5/5^n$

(b) $\sum (\pi/3)^n$

(c) $\sum \frac{1}{n^{\pi/3}}$

(d) $\sum 2^n/n!$

5 Consider the series

$$\sum \frac{\sin(n)}{2^n} .$$

Determine whether or not this series converges. A simple "yes" or "no" answer is not enough; in order to receive credit you must justify your answer carefully.