ath 335, Test 3, Fall (1994)

The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. The problems are worth 20 points apiece; in the case of a problem with multiple sections, the credit will be divided equally between the parts unless otherwise indicated. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (texbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

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1 (a) (5 pts) Define what it means for a function f to be *continuous* at a point a in its domain. (b) (15 pts) Let f be a function which is continuous on the closed interval $[a, b]$. Prove that f is bounded.

2 Let f be a function which is continuous on the closed interval $[a, b]$. Prove that f is uniformly continuous on $[a, b]$.

3 (a) State the Intermediate Value Theorem. (b) Let f be a degree 5 polynomial with leading coefficient 1:

$$f(x) = x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 \ .$$

Use the Intermediate Value Theorem to show that f has at least one real root. (Hint: observe that that $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = \infty$.)

4 (a) Let f be a function on $[1, \infty]$ which is continuous, positive, and decreasing. Draw a picture to illustrate the inequality

$$f(2) + f(3) + \cdots + f(n) \leq \int_1^n f(x)\,dx \ .$$

Next determine which of the following series converge. b $\sum \frac{1}{n(\log n)^2}$ c $\sum (-1)^n \frac{\sqrt{n}}{\log n}$ d $\sum (-1)^n/(n^5 + 10n^2 + 2)$

5 (a) Give an example of a divergent series $\sum a_n$ for which $\sum (a_n)^2$ converges. (b) Give an example of a convergent series $\sum a_n$ for which $\sum (a_n)^2$ diverges.