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Math 335, Final Exam, Fall 1995

The test will be 2 hours in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. Each problem has the point value indicated. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (textbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

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1. 1

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1 (a) (20 pts) Suppose that f is a continuous function from a closed and bounded set H into . Prove that the range of f is also closed and bounded. (b) (10 pts) Let f be the function from the open interval $(0,1)$ into given by $f(x) = 1/x$. Show that there does not exist a continuous function on the closed interval $[0,1]$ which agrees with f on $(0,1)$.

2 (20 pts) Prove that the set of all real numbers is uncountable.

3 (20 pts) Prove that every convergent sequence is a Cauchy sequence.

4 Let (x_n) be a sequence of real numbers. (a) (10 pts) Define $\limsup x_n$. (b) (5 pts) Give an example of a bounded sequence (x_n) with the property that $\limsup x_n$ is not equal to $\liminf x_n$. (c) (5 pts) Give an example of a bounded sequence (x_n) with the property that there exists a number in the sequence which is greater than $\limsup x_n$.

5 (20 pts) Use induction to prove that for any integer $n \geq 1$,

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = n(n+1)(2n+1)/6 \ .$$

6 (a) (5 pts) Define the *interior* of a subset S of . (b) (5 pts) Define what it means for a subset S of to be *open*; define what it means for S to be *closed*. (c) (5 pts) Find a subset of which is neither open nor closed. (d) (5 pts) Find a subset of which is not open, is not closed, and has empty interior.

7 (20 pts) Suppose that $a > 0$ is a real number and that (x_n) is a sequence of points in with $\lim x_n = a$. Prove that there exists a number N such that if $n > N$ then $x_n > 0$.