The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. Each problem has the point value indicated. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (texbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

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"(1)" dom R Q

1 (20 pts) Suppose that a set A of real numbers is both open and closed. Prove that either A is the empty set or $A =$.

2 (20 pts) Prove that the set of real numbers is uncountable.

3 (20 pts) Suppose that r is a real number. Prove (presumably by induction) that the following formula is true for any integer $n \geq 0$:

$$1 + r + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r}$$

4 (a) (5 pts) Define what it means for a subset U of to be a *neighborhood* of a real number x. (b) (5 pts) Define what it means for a subset U of to be *open*. (c) Show that the set of rational numbers is not an open subset of .

5 (a) (11 pts) State the Axiom of Completeness. (b) (9 pts) For each of the following three conditions, either find a subset A of $\mathbb{R}$ which satisfies the condition, or explain why there is no such subset. i A is nonempty and $\sup A$ does not exist. ii $\sup A$ exists and $\sup A \notin A$. iii $\sup A$ exists and $\sup A \in A$.