The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. Each problem has the point value indicated. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (textbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

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1 (20 pts) Prove that any bounded sequence of real numbers has a partial limit (i.e., a convergent subsequence).

2 (20 pts) Prove that any bounded increasing sequence of real numbers converges.

3 (a) (5 pts) Define what it means for a subset A of to be *closed*. (b) (15 pts) Suppose that A is a closed subset of and that (x_n) is a convergent sequence such that $x_n \in A$ for all n. Show that $\lim x_n \in A$.

4 (a) (5 pts) Give the "$\epsilon - -N$" formulation of what it means for a sequence (x_n) to converge to a limit x. (b) (5 pts) Using (a), write down in concrete terms what it means for a sequence (x_n) **NOT** to converge to x. (c) (10 pts). Let $x_n = 1/n^2$. Show carefully using (a) that (x_n) converges to 0.

5 (a) (5 pts) Define what it means for a sequence (x_n) to be a Cauchy sequence. (b) (15 pts) Determine whether or not the following sequences (x_n) are Cauchy sequences: i $x_n = (-1)^n + \dfrac{1}{n}$ ii $x_n = 1 + \dfrac{(1)^n}{n}$ iii $x_n = \log(n)$