The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. Each problem has the point value indicated. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (textbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

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1 (20 pts) Suppose that $f : S \to$ where $S \subset$, and suppose that $a \in (S)$. Prove that the following two conditions are equivalent. a $f(x) \to \alpha$ as $x \to a$. b For every sequence (x_n) in the set $S \setminus \{a\}$ satisfying $x_n \to a$, we have $f(x_n) \to \alpha$.

2 (20 pts) Suppose that S and T are sets of real numbers, that $f : S \to T$ and that $g : T \to$. Suppose that f is continuous at a point $x \in S$ and that g is continuous at the point $f(x)$. Prove that the composition $g \circ f$ is continuous at the point x.

3 (a) (10 pts) Suppose that a is a real number. Define what it means for $\lim_{x\to a} f(x)$ to be $-\infty$. (b) (10 pts) Suppose that α and a are real numbers. Define what it means for $\lim_{x\to a^+} f(x)$ to be equal to α.

4 (20 pts) Starting with your favorite precise formulation of the notion of "limit" (for instance, the ϵ-δ formulation), give a careful argument to show that

$$\lim_{x\to 1}(x^2+1) = 2 \ .$$

5 (a) (10 pts) Suppose that a is a point in the domain of a function f. Give the ϵ-δ formulation of what it means for f to be continuous at a. (b) (10 pts) Suppose that f is a function whose domain is the set of all real numbers. Assume that for any two real numbers x and y, the following inequality is true:

$$|f(x) - f(y)| \leq |x - y| \ .$$

Using (a), show that f is continuous at every point of its domain.