Name: ______________________________

depth0.0cm

MATH 335: Real Analysis I – FALL 2000

EXAM I

This Exam contains 6 problems on 8 sheets of paper, including this front cover. You have to solve problems 1, 2 and three out of four remaining problems. Do all of your work on the paper provided.

Problem	Possible points	SCORE
1	30	
2	25	
3	15	
4	15	
5	15	
6	15	
TOTAL	100	

Problem 1.

Do three of the following five (10 points each).

(a) State the definition of the Cauchy sequence.

(b) State the definition of the divergent sequence.

(c) Give the definition of the intersection of two sets.

(d) State your favorite theorem about limits of sequences.

(e) Give the definition of a countable set.

Problem 2.

Do five of the following seven (5 points each). Explain your answers. If necessary, provide a counterexample.

(a) True or false: if $\{a_n\}$ and $\{b_n\}$ are divergent sequences, then $\{a_n b_n\}$ diverges.

(b) Suppose for every natural n

$$\frac{n-2}{n+1} \leq a_n \leq \frac{n^3+n^2}{n^3+7} .$$

Find $\lim_{n\to\infty} a_n$.

(c) True or false: if $\{a_n\}$ converges, then $\sup\{a_n\} = \lim_{n\to\infty} a_n$

(d) What is the domain of $f+g$, if f and g are functions with domains $Dom(f)$, $Dom(g)$ resp.

(e) True or false: every bounded sequence is convergent.

(f) True or false: a subsequence of a Cauchy sequence is a Cauchy sequence.

(g) True or false: if a is a greatest upper bound of the set S then a belongs to S.

Problem 3. (15 points)

Prove that for any natural n

$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

Problem 4. (15 points)

Give an example of a **divergent** sequence that has **exactly** one limit point.

Problem 5. (15 points)
Let $\{x_n\}$ be defined by

$x_1 = 2,\ x_{n+1} = \frac{3}{4-x_n}$ for $n \geq 1$.
Prove that $\{x_n\}$ has a limit and find it.

Problem 6. (15 points)

Show using the definition of the limit that

$$\lim_{n\to\infty} 2^{-\sqrt{n}} = 0 \ .$$