by -8.9in

Name: ____________________________

depth0.0cm

MATH 335: Real Analysis I – FALL 2000

FINAL EXAM

Problem 1.

Do four of the following five (5 points each).

(a) State the definition of the Cauchy sequence in a metric space (M, ρ).

(e) Let $\{f_n\}$ be a sequence of functions defined on the set $E \subset \mathbb{R}$. Define what it means for f_n to converge **uniformly** to a function f defined on E.

Problem 2.

Do five of the following seven (5 points each). Explain your answers. If necessary, provide a counterexample.

(a) True or false: if a sequence of uniformly continuous functions on $[0, 1]$ converges **point-wise** to a function f, then f is uniformly continuous.

(d) True or false: for $f \in C[a, b]$, $\|f\|_\infty \leq 1 implies$—f(x)— ≤ 1 for all $x \in [a, b]$

(g) If $f \in C^1[1,2]$, $f(1) = 2$ and $|f'(x)| \leq 0.1$ for every $x \in [1,2]$, what is the largest possible value for $f(2)$?

Problem 3. (15 points)

State and prove an important theorem from this class.

Problem 4. (10 points)

Prove that $f(x) = \sqrt{x}$ is uniformly continuous on $[0, \infty)$.

Problem 5. (10 points)
For which $\alpha \geq 0$ an improper integral

$$\int_0^\infty \frac{1}{(x+1)^\alpha} dx$$

exists.

Problem 6. (10 points)

Let f belong to $C^1[a,b]$. Assume that $f'(x) < 0$ for every $x \in [a,b]$. Show that $f(x)$ is strictly decreasing on $[a,b]$.

Problem 7. (10 points)

Find the second Taylor polynomial $T^{(2)}(x, x_0)$ for a function $f(x) = \ln(1 + x^2)$ at $x_0 = 1$.