Math 335 Real Analysis

Exam I

September 21, 2001

Answer all 5 questions. All questions have equal points. Symbols and markings without complete sentences will not be considered as answers.

1. Let f be a function defined on $\mathbf{R}$ and L a real number.

   a) Define what is meant by $f(x) \to L$ as $x \to a$.

   b) Define what is meant by saying f is continuous at $x = a$.

   Let $f(x) = x^2 + 1$.
   Find δ so that $\mid f(x) - f(o) \mid < \epsilon$ when $\mid x - o \mid < \delta$. Your answer will give δ in terms of ϵ.

2. Suppose f_1 and f_2 are two functions defined on $\mathbf{R}$ and $\lim_{x \to a} f_1(x) = L_1$ and $\lim_{x \to a} f_2(x) = L_2$. Define $g(x) = f_1(x) + f_2(x)$. Prove that $\lim_{x \to a} g(x) = L_1 + L_2$.

3. Let a > -1. Use induction to prove $(1 + a)^n \geq 1 + na$ for all positive integers n.

4. Determine each of the following limits:

   i) $\lim_{n\to+\infty}(3+\cos n)n$

   ii) $\lim_{x\to 2^-}\frac{\sqrt{4-x^2}}{\sqrt{6-5x+x^2}}$

   iii) $\lim_{x\to 0}\frac{\sin^2 x}{x^2}$

   You must give the basis of your determination in each case.

5. Let f be a function on $\mathbb{R}$ defined by

$$f(x) = \begin{cases} \frac{2(x^2-9)}{x-3} & x \neq 3 \\ 12 & x = 3 \end{cases}$$

Show that $\mid f(x) - f(3) \mid < \epsilon$ when $\mid x - 3 \mid < \frac{\epsilon}{2}$, for all $\epsilon > 0$.
Is the function of continuous at $x = 3$? Yes or No.