ath 336, Test 1, Spring (1993)

The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. The problems are worth 20 points apiece; in the case of a problem with multiple sections, the credit will be divided equally between the parts unless otherwise indicated. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (texbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

Note: all functions below are *real-valued*.

1

1. 1

"(1)" dom

1 Prove that *the uniform limit of continuous functions is continuous*. More precisely, let (f_n) be a sequence of functions on a set $S \subset \mathbb{R}$, suppose that $f_n \to f$ uniformly on S, and suppose that $S = dom(f)$. Assume that each f_n is continuous at x_0 in S. Prove that f is continuous at x_0.

2 Show that the series

$$\sum_{n=1}^{\infty} \frac{\cos nx}{2^n}$$

converges uniformly on $\mathbb{R}$ to a continuous function.

3 Let (f_n) be the sequence of functions on $\mathbb{R}$ given by $f_n(x) = x/n$. 1 (6 pts) Determine the function f with domain $\mathbb{R}$ which is the pointwise limit of the sequence (f_n). 2 (8 pts) Show that the sequence (f_n) *does not* converge uniformly to f. 3 (6 pts) Let g_n be the restriction of f_n to the interval $[0, 1]$ and let g be the restriction of f to $[0, 1]$. Show that the sequence (g_n) converges uniformly to g.

4 Let f be the function on $\mathbb{R}$ given by $f(x) = x + |x|$. Show that f is not differentiable at $x = 0$.

5 Let f be a function which is defined and differentiable on $\mathbb{R}$. Suppose that $f(0) = 1$, and that $f'(x) < 2$ for all $x > 0$. Use the Mean Value Theorem to show that $f(x) < 2x + 1$ for all $x > 0$.