Math 336, Test 2, Spring (1993)

The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. The first problem is worth 40 points and the others are worth 20 points apiece; in the case of a problem with multiple sections, the credit will be divided equally between the parts unless otherwise indicated. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (texbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

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1 [40 pts] Describe the construction of the integral $\int_a^b f$ and discuss the major properties of the integral.

2 Prove that if f is a function which is continuous on the closed interval $[a, b]$ then f is integrable on $[a, b]$.

3 Determine which of the following functions are integrable on the interval $[0, 2]$. Justify your answers carefully. For each function which is integrable, compute the integral. a [6 pts]

$$f(x) = \{\, 1 \,, for$$

$$0 \le x \le 1$$
x, for $1 < x \le 2$.

$$b[7pts]$$

f(x)= $\{\, 0 \,, if x is a rational number$
1,otherwise.

$$c[7pts]$$

f(x)= $\{\, 1 \,/x^2, if x \ne 0$
0,if $x = 0$.

4 (a) Find the Taylor Series expansion of $f(x) = \sin x$. (b) Indicate why this series converges to $\sin x$ for all $x \in \mathbb{R}$.