Math 336, Test 3, Spring (1993)

The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. The problems are worth 20 points apiece; in the case of a problem with multiple sections, the credit will be divided equally between the parts unless otherwise indicated. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (texbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

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1 Suppose that g is a continuous function on $[a, b]$ which is differentiable on (a, b), and assume that g' is integrable on $[a, b]$. Prove that

$$\int_a^b g' = g(b) - g(a) \quad .$$

2 (a) List the properties satisfied by the distance function d in a metric space. 2 (b) Let B be the set of all bounded sequences $\mathbf{x} = (x_1, x_2, \ldots)$ and define

$$d(\mathbf{x}, \mathbf{y}) = sup\{|x_j - y_j| : j = 1, 2, \ldots\} \quad .$$

Prove that d satisfies the triangle inequality.

3 (a) State the Weierstrass Approximation Theorem as it applies to functions f on a closed interval $[a, b]$. 3 (b) Give an example to show that the conclusion of the theorem is false if the closed interval $[a, b]$ is replaced in the statement by $(-\infty, \infty)$. Justify your answer.

4 (a) Use the identity $\sum_{n=0}^{\infty}(-1)^n x^{2n} = 1/(1 + x^2)$, $x \in (-1, 1)$, to find a power series formula for $arctanx$ that is valid for $x \in (-1, 1)$. 4 (b) Combine this formula with the calculation $tan(\pi/6) = 1/\sqrt{3}$ to obtain an explicit series formula for π.

5 Differentiate the following two functions. a

$$f(x) = \int_0^{x^2} \sin t \, dt$$

b

$$g(x) = x^{\cos x}$$