Math 336, Test 1, Spring (1994)

The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. The problems are worth 20 points apiece; in the case of a problem with multiple sections, the credit will be divided equally between the parts unless otherwise indicated. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (texbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

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1 (a) Define what it means for a sequence (f_n) of functions on a set S to converge uniformly to a function f on S. (b) Let S be the closed interval $[0, 1]$. Give an example of a sequence (f_n) of functions on S such that f_n converges pointwise to a function f on S but (f_n) does **not** converge uniformly to f. Justify your answer.

2 Let (f_n) be a sequence of continuous functions on a set S, and suppose that (f_n) converges uniformly to a function f on S. Prove that f is continuous on S.

3 Let E be a subset of a metric space (S, d). Prove that the set E is closed if and only if it contains the limit of every convergent sequence of points in E.

4 Let $\sum a_n x^n$ be a power series with radius of convergence $R > 0$. a If $0 < R_1 < R$, show that $\sum a_n x^n$ converges uniformly on $[-R_1, R_1]$. b Use (a) to show that the power series converges to a continuous function on $(-R, R)$.

5 a (5 pts) Calculate the radius of convergence of $\sum \frac{n^4}{4^n} x^n$. b (5 pts) Calculate the radius of convergence of $\sum \frac{\pi^n}{n!} x^n$. c (10 pts) Calculate the **interval of convergence** of $\sum \frac{1}{n^2+1} x^n$. Be sure to handle the endpoints carefully. Justify your answer.