Math 336, Test 2, Spring (1994)

The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (texbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

1

**1.** 1

1

"(1)" dom

1 (40 pts) Prove the following theorem.

**Abel's Theorem**. Let $f(x) = \sum_{n=0}^{\infty}$ be a power series with finite positive radius of convergence $R$. If the series converges at $x = R$, then $f$ is continuous at $x = R$. If the series converges at $x = -R$, then $f$ is continuous at $x = -R$.

2 (20 pts) Let $f$ be a continuous function on $[a, b]$ that is differentiable on $(a, b)$ and satisfies $f(a) = f(b)$. Show that there exists at least one $x$ in $(a, b)$ such that $f'(x) = 0$. (This is **Rolle's Theorem**.)

3 (20 pts) **Weierstrass's Approximation Theorem** states that if $f$ is a continuous function on a closed interval $[a, b]$, then there exists a sequence $(p_n)$ of polynomials such that $p_n \to f$ uniformly on $[a, b]$. Show that the conclusion of the theorem fails without the assumption that the interval involved is closed and bounded, by showing that there is no sequence of polynomials which converges uniformly on $\mathbb{R}$ to $f(x) = \cos x$.

4 (20 pts) Let $f(x)$ be the function with domain $\mathbb{R}$ and range $(-\pi/2, \pi/2)$ given by $f(x) = arctanx$. Use the Mean Value Theorem to show that

$$|f(x) - f(y)| \leq |x - y|$$

for all $x$ and $y$ in $\mathbb{R}$.