/bf Math 336, Test 3, Spring (1994)

The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. The problems are worth 20 points apiece; in the case of a problem with multiple sections, the credit will be divided equally between the parts unless otherwise indicated. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (texbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

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1 Let f be a continuous function on $[a,b]$. Prove that f is integrable on $[a,b]$.

2 Let f be a continuous function on $[a,b]$ which is differentiable on (a,b). Suppose that f' is integrable on $[a,b]$. Prove that $\int_a^b f' = f(b) - f(a)$.

3 Let f be a continuous function on $[a,b]$. The **Mean Value Theorem for integrals** describes a particular property of f with respect to integration. a State the Mean Value Theorem for integrals. b Give an example of a discontinuous function f on a closed interval $[a,b]$ for which the conclusion of the Mean Value Theorem for integrals is **false**. Justify your answer. (Hint: consider a step function.)

4 Calculate

$$\lim_{x\to 0} \frac{1}{x} \int_0^x e^{\sin t} dt \ .$$

5 Let f be the function with $f(x) = e^x$ if x is a rational number and $f(x) = 0$ if x is not a rational number. Determine whether or not f is integrable on $[0,1]$. Justify your answer.